R-TR-75-035

STUDY OF MAN-WEAPON REACTION FORCES APPLICABLE TO THE FABRICATION OF A STANDARD RIFLE FIRING FIXTURE

BY

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FINAL REPORT

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A study was conducted of the man-weapon interaction force relationship to define the parameters to be incorporated in the design of a universal small arms test fixture, that simulates man as a flexible mount. This report contains the results of Phase I of the program, which is to provide the engineering data and preliminary drawings required for the fabrication of a prototype test fixture. Phase II will provide for the manufacture and evaluation of the test fixture. (Continued)

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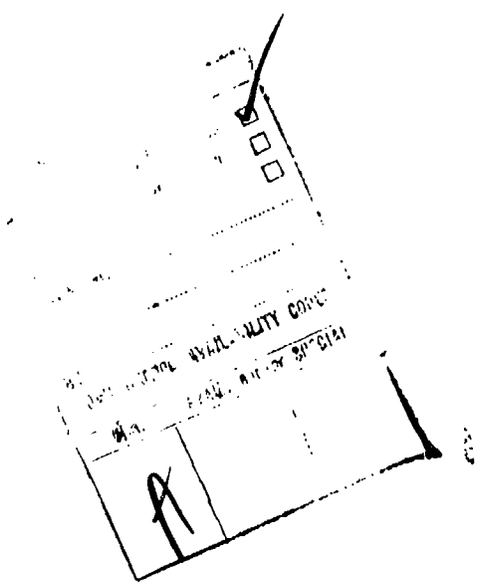
20. To demonstrate the feasibility of designing a universal mount fixture, a three degree of freedom mathematical model was developed that simulates the man and weapon as a coupled dynamical system. A sensitivity analysis was performed on both analog and digital computers to determine which of the system parameters are critical to the design of the mount and to obtain bounds on them. The mathematical simulation was supported by an extensive test firing program, involving shooters of various sizes firing the M16 rifle and the M79 and M203 grenade launchers. In these tests the shoulder reaction force and the dynamical motions of the man-weapon system were recorded. Information provided by the tests was used to estimate some of the critical parameters of the system and to provide a means for validating the model predictions. Result. of this study indicate the feasibility of designing a universal small arms mount based on a two degree of freedom concept that would be capable of testing a variety of small arm weapons.



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1.0 INTRODUCTION

For many years the Army has encountered problems in hard stand test firings of shoulder supported weapons. There are two basic causes for these problems:

a. Test fixtures do not adequately represent actual firing situations, which results in an incongruity between data obtained in test firings and actual field use firings.

b. Test fixtures are not universal, so that a particular weapon might reveal one set of characteristics in one type of fixture and a different set in a second type of fixture. For the above reasons many weapon producers have, in the past, been forced to shut down production because their weapons failed to pass acceptance tests. It is estimated that over $100,000 could have been saved on the M16 rifle program if only one of the many production shut downs due to weapon malfunctioning was eliminated.

The objectives of this report are to determine the feasibility of designing a small arms mount, that adequately simulates the mount reaction force for an actual shoulder supported firing configuration, and to propose a design concept of a mount fixture and arrive at the critical design parameters for the fixture.

To show the feasibility of designing a mounting fixture, a prototype mathematical model was developed that simulates the man and weapon as a coupled dynamical system. The derivation of the model equations is presented in Section 2.0.

Successfully modeling man as an integral part of a weapon system rests upon two important facts. First, the dynamical motions that occur in a typical shooting situation are characterized by small angle oscilliatory motions of the human operator about an initial aiming position. Second, the applied breech pressure force consists of a periodic sequence of impulses. Man's neuro-muscular reaction time is slow compared with the weapon firing rate and his ability to think and actively respond to these force inputs does not influence the dynamics of the system until after a significant amount of time has elapsed. An initial passive response phase of motion is, therefore, a characteristic of the man-weapon interaction problem.

Before listing the specific modeling assumptions, a discussion of the exact nature of human dynamical response is presented. The human body is a nonhomogeneous composition of body segments with multiple degrees of freedom. In addition to being nonhomogeneous, each body segment is deformable because of blood flow and muscle action; furthermore, the human neuromuscular system behaves as a servo control mechanism to force inputs. Some pecularities such as blood flow have little affect on the dynamical behavior of the human body. The dynamical representations of the body segments and the representation of the shooter's behavior as a servo control mechanism is treated by making certain mechanical assumptions.

Because of the overall complexity of the human body, mathematical models of mechanical systems involving human body interactions must pertain to rather specific configurations. Instead of modeling the man-weapon interaction problem for all types of firing positions, a more suitable analytical approach is to model one specific firing position. In this way the kinematical constraints become easier to prescribe and the resulting equations of motion become less complex. Usually two or three firing positions are standard for small arm weapons. The vertical standing position, with the rifle held up against the right shoulder, was chosen for this analysis primarily because the motions of the man and weapon in this position can be adequately represented with only three independent degrees of freedom.

For convenience the time history of motion of the man-weapon system is separated into a set of distinct phases. In particular, the human operator's neuromuscular response is separated into a passive phase followed by an active phase. Breech pressure forces for the M16 rifle consist of a series of impulses of approximately one millisecond duration, which repeat periodically approximately every 80 milliseconds. Because the human body is not capable of reacting actively to force inputs during the first 150-200 ms, this initial time interval is characterized by a passive response of the human operator. Afterwards, the neuromuscular system, which acts somewhat as a servo control mechanism, can significantly influence the weapon motion; however the physical effects of the man's active neuromuscular response generally are not noticeable until after 300 milliseconds have elapsed.

Much insight, in the problem of modeling man-weapon interactions, was provided by observations of high speed photographic films that were obtained from tests conducted at the Harry Diamond Laboratory. In these films, both the top view and the side view of a man firing the M16A1 rifle in automatic fire were recorded. A stationary grid was placed behind the test subjects to provide an inertial reference system. The rifle was held by the test subjects in each of three separate firing positions:

a. off the hip

b. on the right shoulder with the body in a vertical standing position

c. on the right shoulder with the body in a slanted position

The vertical standing position was selected for the theoretical analysis.

Observations of the films helped to establish the predominant degrees of freedom of the system and the time history of motion in each coordinate. Additional high speed photographic data were obtained by Mr. Thomas Hutchings for the M79 and M203 grenade launchers. This data provided the time history motions for the higher impulse weapons (the M79 and M203 grenade launchers generate an impulse of about 2.5 lb-sec compared with 1.2 lb-sec for the M16 rifle). Additional data was obtained for the M16 rifle at the Keith L. Ware Simulation Laboratory, Rock Island Arsenal. A load cell device was used to measure the transmitted shoulder mount force and two displacement transducers were used to obtain the weapon rotation. These experiments are discussed more fully in Section 3.0. The data from these experiments provided considerable insight on how to construct the analytical model and also provided a means for estimating some of the unknown system parameters.

Section 4.0 contains the results of a sensitivity analysis that was performed with the man-weapon interaction model. As a result of this analysis, two prototype small arms mount designs are recommended. The conclusions and recommendations of this study are presented in Section 5.0.

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2.0 DERIVATION OF THE BIOMECHANICAL EQUATIONS OF MOTION FOR SHOULDER FIRED SMALL ARMS

2.1 Test Fixture Design Procedure

The design specifications for the universal small arms mount fixture were obtained from analytical simulations of the biomechanical interaction problem. That is, the design configuration and design parameters of the mount are based on results of a dynamic analysis of the man-weapon interaction problem. The specific steps followed in this procedure are listed below:

a. Determine, through observations of high speed film data, the minimum required number of independent degrees of freedom needed to represent the biomechanical interaction forces for shoulder supported small arm weapons.

b. Develop a biodynamical model of the man-weapon system and determine the sensitivities, of mount force and of the pitching motion of the weapon, to variations in the various biomechanical system parameters.

c. Obtain a preliminary design for a universal small arms test fixture based on the biomechanical analog and obtain bounds on the critical design parameters from results of the sensitivity analysis.

2.2 High Speed Photographic Test Results

In order to determine the number of independent degrees of freedom that are associated with the man-weapon interaction problem, several high speed photographic film tests were performed for shoulder supported firings of the M16A1 rifle and the M79 and M203 grenade launchers. These tests involved several shooters of various weights and heights so that the effects of variations in human mount characteristics would be observed. The data extracted from these tests include the transmitted recoil force and motions of the weapon and the shooter. The shoulder support firing configuration was selected for the tests, because the weapon mount forces are easier to measure and the biodynamical motions are less complex than for other standard firing configurations, such as the off-hip configuration.

Observations of the high speed photographic film data led to the development of a man-weapon biodynamics model. The films recorded the test subjects in both the top and side views. The top view was obtained by a 45° inclined mirror located directly above the test subject. During the tests, the shooters fired five rounds from the M16A1 rifle in single shot, semi-automatic, and burst modes, and fired single shots from the grenade launchers at various range settings. Observations of the film sequences for these tests indicated which rotational and translational motions of the weapon and shooter are predominant.

In addition to the translatory motion of the weapon toward the shoulder, two predominant motions of the man-weapon system were observed. These motions include:

a. A rotation in the vertical plane of the upper torso of the man, who initially (for approximately 0.3 second duration) pivoted about his hips. Afterwards the man appears to consciously react to the weapon recoil and his motion becomes more complex.

b. A rotation in the vertical plane of the weapon and the shooter's arms, pivoting about the shoulder. The top view revealed a negligible amount of yawing of either the shooter or the weapon compared with the vertical pitching rotations. Selected angles and position coordinates were measured for each film sequence using the Vanguard Motion Analyzer at the University of Iowa Hospital Biomechanics Laboratory in Iowa City, Iowa.

Results of the high speed photographic film tests reveal a period of relative inactivity in the pitching rotations for approximately the first 20-50 milliseconds after the commencement of each shot. This delay effect is probably caused by the compression of weapon padding and soft body tissue as the weapon translates toward the shoulder. After the weapon is fully compressed against the shoulder the remaining kinetic energy of the weapon induces the rotational pitching motions of both the man and the weapon. Yawing motion apparently was minimized by the stance of the man, as the weapon is held almost parallel with the breadth of his chest during firing. The net torque produced about the man's vertical yaw axis is therefore small.

2.3 Man-Weapon Model Assumptions

A mathematical model simulating the man-weapon biodynamical interactions from weapon recoil was then developed based on information supplied by the film data and on biomechanical data supplied by various sources. Figure 1 contains a schematic representation of the model. The X_1, X_2, X_3 coordinate system is a fixed system with the origin at the man's hip. Coordinate system Y_1, Y_2, Y_3 is attached to the right shoulder pivot and rotates with the weapon. The Y_2 axis is parallel to the gun barrel centerline. Variable x locates the center of mass of the weapon combined with the man's arms. Variable θ measures the absolute pitch of the man in the X_2 - X_3 plane and variable ϕ measures the pitch of the weapon relative to the man's trunk. The total pitch of the weapon is therefore the sum of angles θ and ϕ.

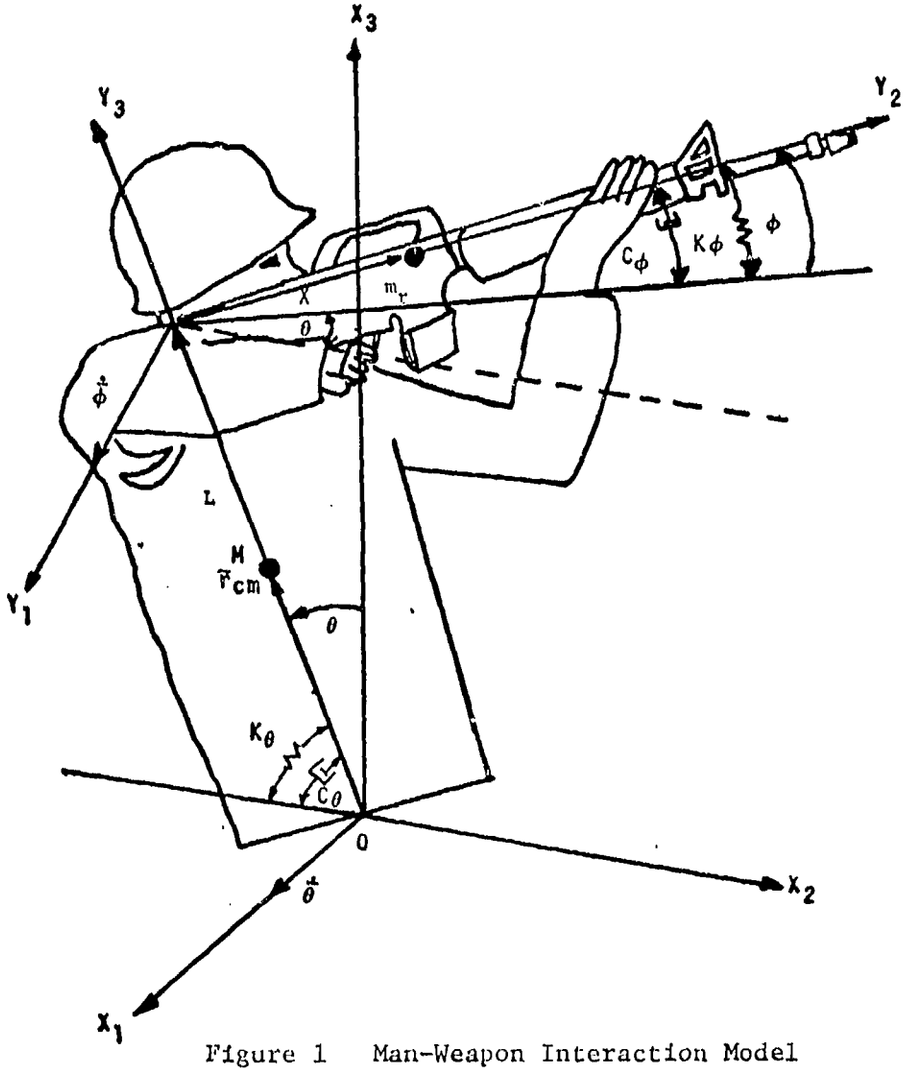

Figure 1 Man-Weapon Interaction Model

Because the development of the biodynamics model involves, in part, a mechanical description of the human body, several simplifying assumptions were required in order to arrive at a fairly simplified but realistic representation of the system. There is no simple or easy approach to modeling human body dynamics and very little useful data is available on biomechanical properties of the human body. The feasibility of successfully modeling man as an integral part of a weapon system rests upon several important considerations, which are discussed below:

(1) The breech pressure force for small arm weapons - rifles and grenade launchers - is impulsive. A typical breech pressure force curve for the M16A1 rifle is illustrated in Figure 2. Normally the M16A1 generates approximately 1.2 lb-sec impulses of about one millisecond duration that repeat periodically every 80 milliseconds.

(2) During the initial portion of a burst, the shooter responds passively to transmitted mount forces. Thus, even though he may anticipate the impulsive recoil force and prepares his muscles accordingly, there is an initial time interval during which the shooter is unable to respond actively to the pulse.

The duration of this passive phase depends primarily on the shooter's neuromuscular reaction characteristics. Moreover, the active response of the shooter intuitively should not have much influence on the time history of the transmitted recoil force. One might expect, however, that the active human control response, after approximately 0.3 seconds or so, might have a significant effect on the angular positioning of both the weapon and the man's upper torso.

c. Observations of the high speed photographic film data reveal that, during the passive response phase, the shooter's hips remain stationary; consequently only the upper torso and arm segments are represented in the biodynamics model.

d. High speed photographic films reveal that the maximum angular pitching motions of the man's upper torso and the weapon are generally less than 10°. A small angle approximation is, therefore, applicable in the formulation of the biodynamical equations of motion. Small angle approximations are particularly useful for generating problem solutions on the analog computer.

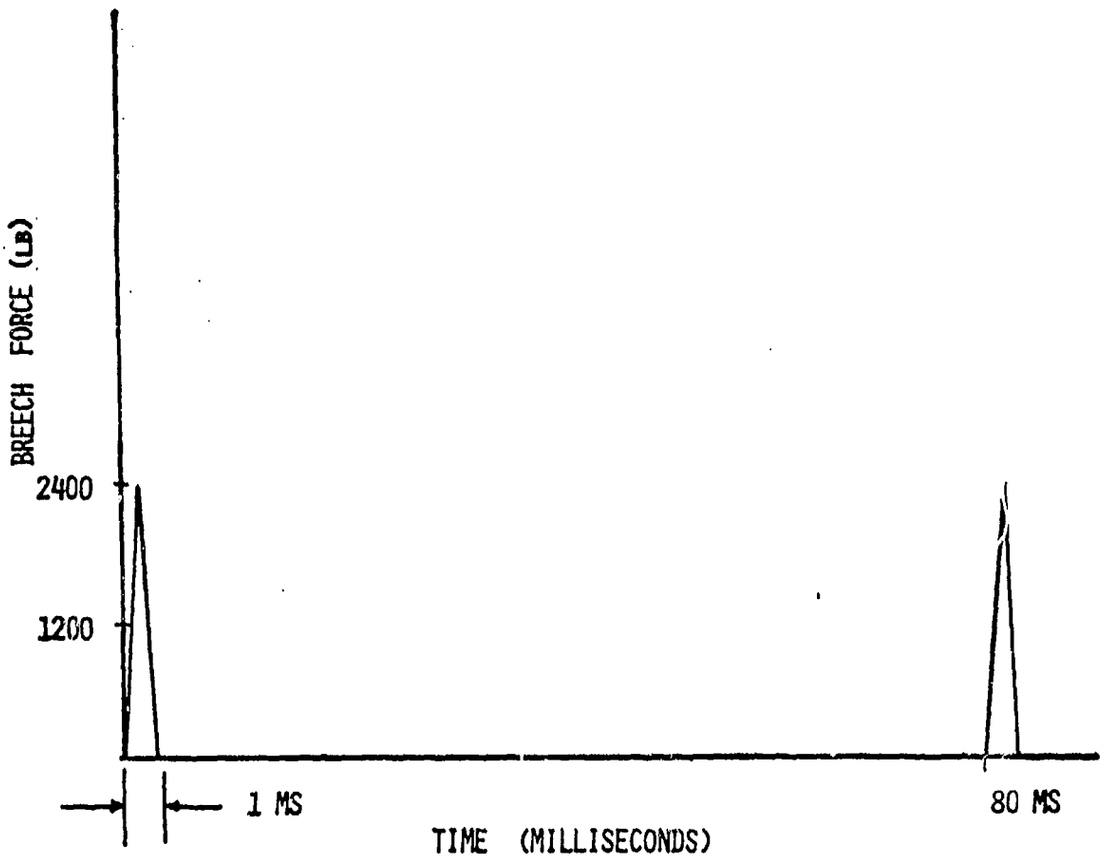

Figure 2 M16 Breech Force vs Time

e. The human body can be represented dynamically as a system of linked rigid bodies that are internally stable and homogeneous; furthermore, each rigid body can be represented by simple geometric forms (i.e. elliptical cylinder, frustrum of a cone, etc). These approximations of the human body have been used in the works published by the investigators listed in References [1,2,3].

2.4 Historical Background on Human Biodynamical Research

The lumped mass approach to modeling the dynamics of the human body requires the specification of segment masses, moments of inertia and centers of mass. Active interest in determining the segment characteristics of the

[1]Whitsett, C.E., "Some Dynamic Response Characteristics of Weightless Man," Master of Science Thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, AMRL-TR-63-18, AD 412541, 1962

[2]McCrank, J.M. and Seger, D.E., "Torque Free Rotational Dynamics of a Variable Configuration Body (Application to Weightless Man)," Master of Science Thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, AD 610239, 1964

[3]McHenry, R.R. and Naab, K.N., "Computer Simulation of the Automobile Crash Victim in a Frontal Collision--A Validation Study," Cornell Aeronautical Laboratory, Inc., Buffalo, New York, CAL Report No. 4B-2126-V-1R, 1966

human body has been undertaken by numerous investigators. Braune and Fisher [4] in 1889 performed experimental research on three cadavers and published a comprehensive study of weights, volumes, and centers of mass of the body and its segments. Later, in 1906, Fisher [5] experimentally determined the moments of inertia of segments from a single cadaver. Since that time interest in this subject diminished until rather recently. Dempster [6], in 1955, published a study on human biomechanics that contains experimental data taken from eight cadavers.

The specific data presented in this study includes segment weights, moments of inertia, densities, center of mass locations and volumes. Using data compiled by Dempster, Braune and Fisher, Barter[7] in 1957 prepared a series of regression equations for predicting body segment weights from total body weight. These equations have been used extensively by biomechanical engineers and designers. In 1963 Santschi[8] and his co-workers reported on moments of inertia and centers of mass of sixty-six live test subjects. Santschi attempted to answer the pertinent question of whether or not body segment parameters can be predicted, to a reasonable degree of accuracy, from anthropometric dimensions. He discovered a high correlation factor of segment centers of mass and moments of inertia with an individual's anthropometric dimensions.

2.5 System Kinetic Energy

The formulas that were used to predict the physical characteristics of the man's upper torso and arm segments are listed in Appendix A. Since the man-weapon interaction problem has been reduced to three independent degrees of freedom, the segment masses corresponding to the man's upper torso (i.e. the head and torso) are combined to form a single rigid body

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[4]Braune, W. and Fischer, O., "The Center of Gravity of the Human Body as Related to the German Infantryman," Leipzig, ATI 138452, 1889

[5]Fischer, O., "Theoretical Fundamentals for a Mechanics of Living Bodies with Special Applications to Man as Well as to Some Processes of Motion of Machines," B.G. Tubner, Berlin, ATI 153668, 1906

[6]Dempster, W.T., "Space Requirements of the Seated Operator," Wright Air Development Center, TR-55-159, Wright-Patterson Air Force Base, Ohio, AD 87892, 1955

[7]Barter, J.T., "Estimation of the Mass of Body Segments," Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, TR-57-260, AD 118222, 1957

[8]Santschi, W.R., DuBois, J., and Omoto, C., "Moments of Inertia and Centers of Gravity of the Living Human Body, Aerospace Medical Research Laboratories, Wright-Patterson Air Force Base, Ohio, AMRL-TDR-63-66, AD 410451, 1963

and the segment masses corresponding to the man's arm segments and hands and the weapon are combined to form a second rigid body. The rigid body comprising the trunk and the head is allowed to rotate in the vertical plane about the man's hip, while the rigid body comprising the man's arms and the weapon is allowed to translate in the direction of the recoil force and rotate vertically about the shoulder hinge. Any bending of the man's arms during a firing event is considered small and is neglected.

Referring to Figure 1, the kinetic energy of the system is given by

$$T = \frac{1}{2}\left[I_{cm}\,\dot{\theta}^2 + I_r\,(\dot{\theta}+\dot{\phi})^2 + M\,\dot{\bar{r}}_{cm}\cdot\dot{\bar{r}}_{cm} + m_r\,\dot{\bar{r}}\cdot\dot{\bar{r}}\right] \qquad (2\text{-}1)$$

where

I_{cm} = Moment of inertia about the center of mass of the head-trunk rigid body.

I_r = Moment of inertia about the center of mass of the arm-weapon rigid body.

M = Mass of the head-trunk rigid body.

m_r = Mass of the arm-weapon rigid body.

$\dot{\bar{r}}_{cm}$ = Time derivative of the center of mass location of the head-trunk rigid body, and,

$\dot{\bar{r}}$ = Time derivative of the center of mass location of the arm-weapon rigid body.

In the X_1, X_2, X_3 frame, the components of vectors $\bar{r}_{cm}$ and $\bar{r}$ are

$$\bar{r}_{cm} = r_{cm}\,(-\sin\theta\,\hat{x}_2 + \cos\theta\,\hat{x}_3) \qquad (2\text{-}2)$$

$$\bar{r} = [x\cos(\theta+\phi) - L\sin\theta]\,\hat{x}_2 + [x\sin(\theta+\phi) + L\cos\theta]\,\hat{x}_3 \qquad (2\text{-}3)$$

Taking the time derivatives of expressions (2-2) and (2-3) and substituting into equation (2-1) leads finally to result

$$T = \left\{\frac{1}{2}\left(I_{cm} + M\,r_{cm}^2\right)\dot{\theta}^2 + I_r\,(\dot{\theta}+\dot{\phi})^2 + m_r\left[(\dot{x} - L\,\dot{\theta}\cos\phi)^2 + (x\,\dot{\phi} + (x + L\sin\phi)\,\dot{\theta})^2\right]\right\} \qquad (2\text{-}4)$$

2.6 System Potential Energy, Dissipative Mechanisms, And Generalized Forces

In order to obtain an expression for the system potential energy, the passive response of the man's muscles must be taken into account. Prior to firing, the shooter's muscles are in a state of preloading to support the weapon and to balance the forces of gravity on both the man and weapon. After the commencement of firing, the approximation is made that passive restoring forces, produced by the various muscles in the upper torso and arms, can be represented by a system of linear springs and dashpots. This approximation is justified based on the small orders of magnitude of the angular deformations resulting from weapon recoil torques. Torsional springs and dashpots are, therefore, included in the model to resist motions in each of the two rotational degrees of freedom. The rearward translatory motion of the weapon initially causes the fleshy padding on the shoulder to compress. Any further rearward displacement of the rifle results in the stretching of the muscles and ligaments that attach to the bone structure of the shoulder. Resistance offered by these muscles and ligaments is modeled by a relatively strong spring and dashpot, while the initial soft compression of the shoulder is modeled by a relatively weak spring. Having represented the elastic and viscous damping characteristics of the model, the system potential energy, V, then becomes

$$
\begin{aligned}
V = {} & \frac{1}{2} k_\theta (\theta - \theta_e)^2 + \frac{1}{2} k_\phi (\phi - \phi_e)^2 \\
& + \begin{cases} \frac{1}{2} k_{x_w} (x - x_e)^2 & \text{for } x \geq x_s \\ \frac{1}{2} k_{x_s} (x - x_s)^2 + k_{x_w} (x_s - x_e) \left[x - \frac{1}{2}(x_s + x_e)\right] & \text{for } x < x_s \end{cases} \\
& + M g r_{cm} (\cos\theta - \cos\theta_0) + m_r g [L(\cos\theta - \cos\theta_0) \\
& + x \sin(\theta + \phi) - x_0 \sin(\theta_0 + \phi_0)]
\end{aligned}
\tag{2-5}
$$

where

k_θ, k_ϕ, k_{x_w}, k_{x_s} = system spring constants

θ_o, ϕ_o, x_o = initial conditions for variables θ, ϕ, and x respectively

θ_e, ϕ_e, x_e = static equilibruim spring positions

g = gravity constant

and

x_s = distance from the shoulder at which the shoulder spring stiffness changes value.

Because the only dissipative mechanisms in the model are viscous dampers, a dissipative force potential, P, can be defined and is given by

$$P = \frac{1}{2} c_x \dot{x}^2 + \frac{1}{2} c_\theta \dot{\theta}^2 + \frac{1}{2} c_\phi \dot{\phi}^2 \qquad (2\text{-}6)$$

where c_x, c_θ, and c_ϕ are the damping constants for the variables x, θ, ϕ, respectively.

Referring to Figure 3, the generalized forces produced by the breech pressure force, F(t), are

$$Q_x = - F(t) \qquad (2\text{-}7a)$$

$$Q_\theta = (\bar{r}_{D/A} \times \bar{F}) \cdot \hat{x}_1$$

and

$$Q_\phi = (\bar{r}_{D/B} \times \bar{F}) \cdot \hat{y}_1$$

where

$$\bar{r}_{D/A} = [-L \sin\theta + x \cos(\theta + \phi) - \delta \sin(\theta + \phi)]\, \hat{x}_2 + [L \cos\theta + x \sin(\theta + \phi) + \delta \cos(\theta + \phi)]\, \hat{x}_3$$

$$\bar{r}_{D/B} = x\, \hat{y}_2 + \delta\, \hat{y}_3$$

$$\bar{F} = -F(t)\, \hat{y}_2 = -F(t) [\cos(\theta + \phi)\, \hat{x}_2 + \sin(\theta + \phi)\, \hat{x}_3]$$

Expanding the above expressions for Q_θ and Q_ϕ yields finally the results

$$Q_\theta = (\delta + L \cos\phi)\, F(t) \qquad (2\text{-}7b)$$

and

$$Q_\phi = \delta \cdot F(t) \qquad (2\text{-}7c)$$

2.7 Dynamical Equations of Motion

The dynamical equations of motion of the man-weapon interaction problem were derived using the Lagrangian formulation,

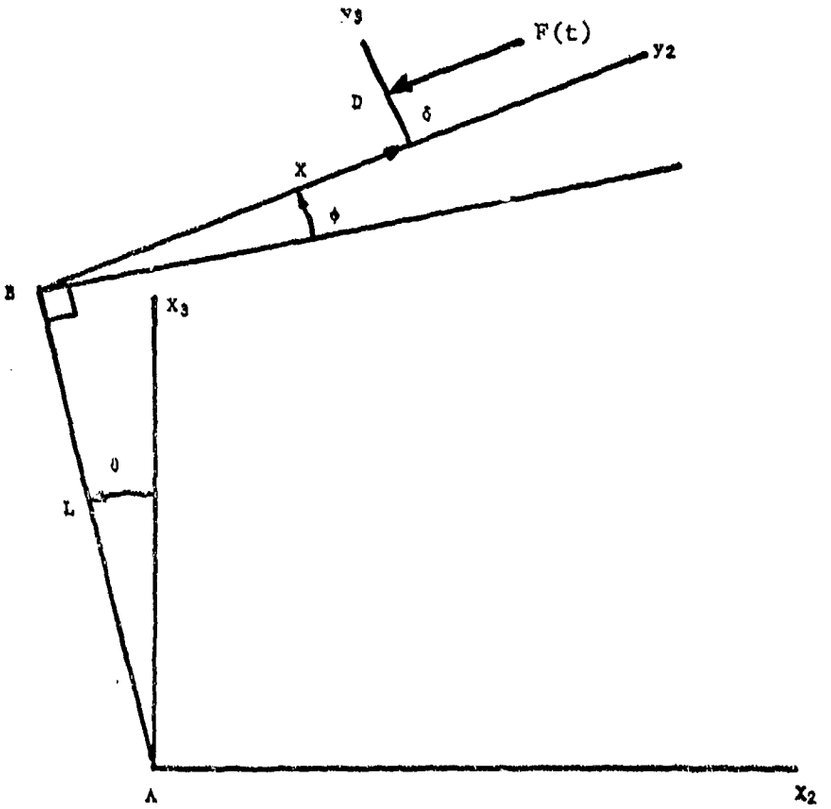

Figure 3 Generalized Force Diagram

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + \frac{\partial P}{\partial \dot{x}} = Q_x \tag{2-8a}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} + \frac{\partial P}{\partial \dot{\theta}} = Q_\theta \tag{2-8b}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} + \frac{\partial P}{\partial \dot{\phi}} = Q_\phi \tag{2-8c}$$

where $L \triangleq T - V$ is the Lagrangian potential function. Substituting equations (2-1) to (2-7) into equations (2-8) and rearranging terms leads finally to the following matrix equation:

$$\begin{bmatrix} a_{11} & -a_{21} & 0 \\ -a_{21} & a_{22} + a_{33} & a_{23} \\ 0 & a_{23} & a_{33} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{\phi} \end{Bmatrix} = \begin{Bmatrix} f_x \\ f_\theta \\ f_\phi \end{Bmatrix} \tag{2-9}$$

where

$$a_{11} = m_r$$

$$a_{21} = m_r L \cos \phi$$

$$a_{22} = I_{cm} + M r_{cm}^2 + m_r L^2 + 2 m_r L x \sin \phi$$

$$a_{23} = a_{33} + m_r L x \sin \phi$$

$$a_{33} = I_r + m_r x^2$$

$$f_x = - F(t) + m_r x (\dot{\theta} + \dot{\phi})^2 + m_r \dot{\theta}^2 L \sin \phi$$

$$- c_x \dot{x} - m_r g \sin (\theta + \phi)$$

$$- \begin{cases} k_{x_w} (x - x_e) , & \text{for } x \geq x_s \\ k_{x_s} (x - x_s) + k_{x_w} (x_s - x_e) , & \text{for } x < x_s \end{cases}$$

$$f_\theta = (\delta + L\cos\phi)\,F(t) - c_\theta\,\dot{\theta} - 2\,m_r\,\dot{x}\,(\dot{\theta}+\dot{\phi})\,(x + L\sin\phi) - m_r\,L\,x\,\dot{\phi}\,(2\dot{\theta}+\dot{\phi})\cos\phi - k_\theta\,(\theta - \theta_e) + M\,g\,r_{cm}\sin\theta + m_r\,g\,[L\sin\theta - x\cos(\theta+\phi)]$$

$$f_\phi = \delta\cdot F(t) - 2\,m_r\,x\,\dot{x}\,(\dot{\theta}+\dot{\phi}) + m_r\,L\,x\,\dot{\theta}^2\cos\phi - c_\phi\,\dot{\phi} - k_\phi\,(\phi - \phi_e) - m_r\,g\,x\cos(\theta+\phi)$$

The static equilibrium spring positions x_e, θ_e, and ϕ_e are determined from the requirement that initially

$$f_x = f_\theta = f_\phi = 0$$

if $F(0) = 0$ and the initial velocity components are zero. These conditions imply

$$f_x = -F(t) + m_r\,x\,(\dot{\theta}+\dot{\phi})^2 + m_r\,\dot{\theta}^2\,L\sin\phi - c_x\,\dot{x} - m_r\,g[\sin(\theta+\phi) - \sin(\theta_o+\phi_o)] - \begin{cases} k_{x_w}\,(x - x_o), & \text{for } x \geq x_s \\ k_{x_s}\,(x - x_s) + k_{x_w}\,(x_s - x_o), & \text{for } x < x_s \end{cases}$$

$$f_\theta = (\delta + L\cos\phi)\,F(t) - 2m_r\,\dot{x}\,(\dot{\theta}+\dot{\phi})\,(x + L\sin\phi) - m_r\,L\,x\,\dot{\phi}\,(2\dot{\theta}+\dot{\phi})\cos\phi - c_\theta\,\dot{\theta} - k_\theta\,(\theta - \theta_o) + (Mr_m + m_rL)\,g\,(\sin\theta - \sin\theta_o) - m_rg\,x\,[\cos(\theta+\phi) - \cos(\theta_o+\phi_o)]$$

and

$$f_\phi = \delta\,F(t) - 2\,m_r\,x\,\dot{x}\,(\dot{\theta}+\dot{\phi}) + m_rL\,x\,\dot{\theta}^2\cos\phi - c_\phi\,\dot{\phi} - k_\phi\,(\phi - \phi_o) - m_rg\,x\,[\cos(\theta+\phi) - \cos(\theta_o+\phi_o)].$$

2.8 Solution Techniques

Two methods were used to solve the matrix equation (2-9). In the first method, the system of equations were solved numerically on a digital

computer by the Runge-Kutta method. In order to apply the Runge-Kutta method, equation (2-9) is premultiplied by the inverse of the coefficient matrix and the resulting system of three second order differential equations is transformed into a system of six first order differential equations. This results in the set of matrix equations given below:

$$\frac{d}{dt}\begin{Bmatrix}\dot{x}\\ \dot{\theta}\\ \dot{\phi}\end{Bmatrix} = \frac{1}{\Delta}\begin{bmatrix} b_{11} & b_{12} & b_{13}\\ b_{12} & b_{22} & b_{23}\\ b_{13} & b_{23} & b_{33}\end{bmatrix}\begin{Bmatrix} f_x\\ f_\theta\\ f_\phi\end{Bmatrix} \qquad (2\text{-}10a)$$

and

$$\frac{d}{dt}\begin{Bmatrix} x\\ \theta\\ \phi\end{Bmatrix} = \begin{Bmatrix}\dot{x}\\ \dot{\theta}\\ \dot{\phi}\end{Bmatrix} \qquad (2\text{-}10b)$$

where

$$\Delta = \det [a]$$

$$= (a_{33}^2 - a_{23}^2 + a_{22}\, a_{33})\, a_{11} - a_{21}^2\, a_{33},$$

$$= (I_{cm} + M\, r_{cm}^2)\, m_r\, (I_r + m_r\, x^2) + I_r\, m_r^2\, L^2 \sin^2\phi,$$

$$b_{11} = (I_{cm} + M\, r_{cm}^2 + m_r\, L^2)\, I_r + m_r\, x^2\, (I_{cm} + Mr_{cm}^2 + m_r L^2 \cos^2\phi)$$

$$b_{12} = (I_r + m_r\, x^2)\, m_r\, L \cos\phi$$

$$b_{13} = -\,[I_r + m_r\, (x + L \sin\phi)\, x]\, m_r\, L \cos\phi,$$

$$b_{22} = m_r\, (I_r + m_r\, x^2),$$

$$b_{23} = -\, m_r\, [I_r + m_r\, x\, (x + L \sin\phi)],$$

and

$$b_{33} = m_r\, [I_{cm} + M\, r_{cm}^2 + I_r + m_r\, (x + L \sin\phi)^2].$$

The solutions to equations (2-10) were obtained by separating the weapon cycle into two time intervals. For the one millisecond time interval, on

which the impulsive breech pressure force is applied, equations (2-10) are integrated with the assumption that the positional coordinates are constant. Thus let

$$I_F \triangleq \int_{t_k}^{t_{k+1}} F(t)dt$$

where t_k = starting time for the shot and $t_{k+1} = t_k + 0.001$. The integration of equations (2-10) yields a set of initial conditions for the start of the second time interval,

$$\begin{Bmatrix} \dot{x}(t_{k+1}) \\ \dot{\theta}(t_{K+1}) \\ \dot{\phi}(t_{k+1}) \end{Bmatrix} = \begin{Bmatrix} \dot{x}(t_k) \\ \dot{\theta}(t_k) \\ \dot{\phi}(t_k) \end{Bmatrix} + \frac{I_F}{\Delta(t_k)} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \Bigg|_{t=t_k} \begin{Bmatrix} -1 \\ \delta+L\cos\phi \\ \delta \end{Bmatrix} \quad (2\text{-}11a)$$

and

$$\begin{Bmatrix} x(t_{k+1}) \\ \theta(t_{k+1}) \\ \phi(t_{k+1}) \end{Bmatrix} = \begin{Bmatrix} x(t_k) \\ \theta(t_k) \\ \phi(t_k) \end{Bmatrix} \quad (2\text{-}11b)$$

For the remainder of the weapon cycle (i.e. until the commencement of the next shot), the equations (2-10) are solved by the Runge-Kutta method (with $F(t) = 0$) and equations (2-11) provide the initial conditions. This solution procedure is then repeated for each of the remaining shots in the burst. A listing of the computer program used to obtain the numerical solutions to the problem is provided in Appendix B.

The digital computer solutions to the biodynamics problem were obtained for a range of biomechanical parameters corresponding to a small, average and large shooter, and for a range of weapon parameters corresponding to the M16A1 rifle and the M79 and M203 grenade launchers. A sensitivity analysis was performed to obtain estimates of the unknown spring and dash-

pot parameters, for several of the weapon and shooter configurations. The results of this analysis are discussed in Section 4.0.

In the second solution method, the system of equations (2-9) were first simplified by small angle approximations, scaled and then programmed on an analog computer. Analog techniques are suitable for performing a parametric sensitivity analysis, because each system parameter can be conveniently changed by adjusting a pot setting and the solutions are displayed immediately on a recording device. Further details regarding the analog simulation are provided in Appendix C.

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3.0 MAN-WEAPON INTERACTION EXPERIMENTS

3.1 Description of Experimental Tests

In order to gather the critical information needed to analytically describe the man-weapon interaction problem, three separate series of experiments were performed that involved test firings with several shooters of various sizes and various weapon types. Data gathered from these tests include:

a. M16 data obtained from the Harry Diamond Laboratory using high speed photographic equipment to record time histories of rotations and displacements of body segments and of the weapon.

b. M79 and M203 grenade launcher data obtained at the Harry Diamond Laboratory test facility using high speed photographic equipment to record rotations and displacements of the weapon and the man.

c. M16 data from the Keith L. Ware Simulation Laboratory, obtained using a fixture between the shoulder and the weapon to measure force reactions at the shoulder and two displacement transducers located at the front and rear of the weapon to measure rotations of the weapon.

In both photographic data experiments at the Harry Diamond Laboratory, the shooters stood in front of a background of vertical and horizontal grid lines. These grid lines formed a fixed reference frame from which relative motions of the man and weapon were measured. Both top and side views were photographed. The top view was recorded by use of an inclined mirror placed directly above the test subject (see Figure 4). The film speed of 1100 frames/second was sufficiently fast to record the ejection of muzzle smoke after each shot from the M16 rifle. Thus the time of occurrence of each shot was measured for the five shot burst. Fixed reference points were selected at the intersections of stationary grid lines on the side and top views. For each frame, the positions of several selected points on the man and weapon were measured with respect to the selected reference points using a Vanguard Motion Analyzer. A computer program was then used to process and convert the raw data into values of angular displacements of the weapon and man.

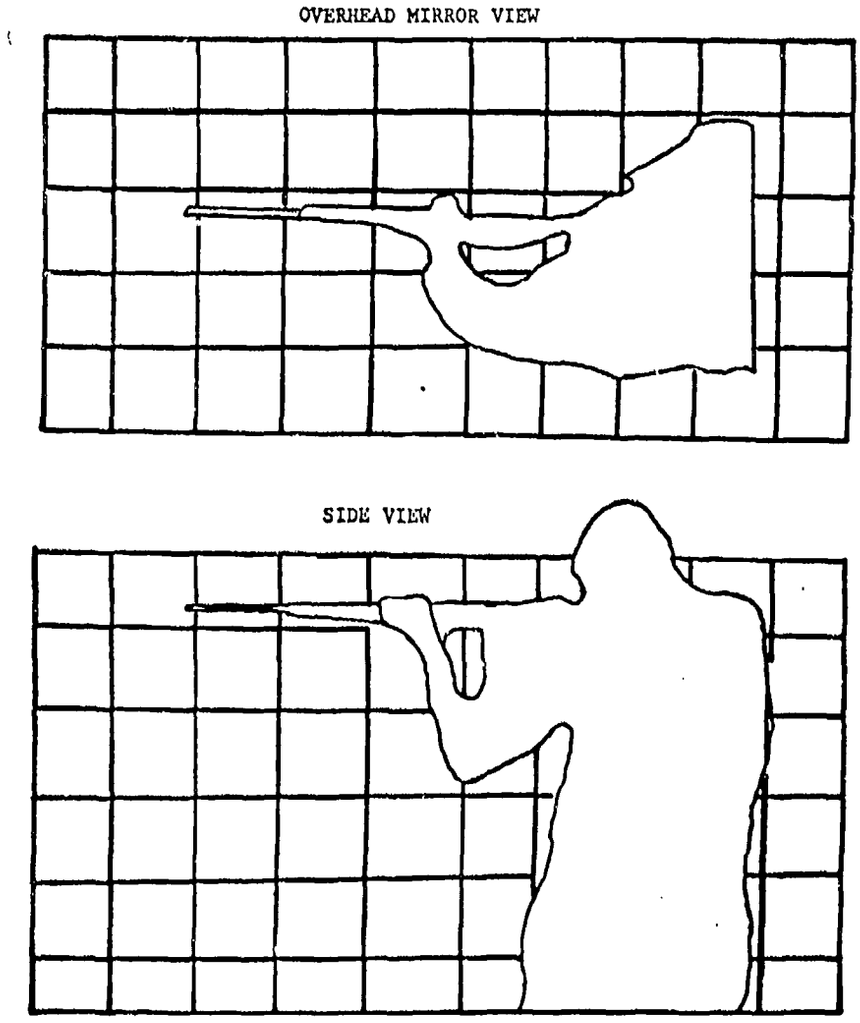


Figure 4 M16 High Speed Photographic Experiment

3.2 M16A1 Test Results

Some of the important observations and interpretations of the M16 data are discussed below. Two major angular motions of the weapon system were observed in addition to the translatory motion of the rifle towards the shoulder. They are:

a. An angular deflection of the man's trunk that pivoted about his hip. This motion is referred to as pitch of the man.

b. An angular pivoting of the arms and rifle about the man's shoulder. This motion is referred to as pitch of the rifle.

A plot of the two pitch coordinates versus time is shown in Figure 5. A period of relative inactivity is noticable in the pitch of the man just after the first and second shots. During this period of inactivity, the weapon most likely is compressing against the soft muscular portion of the shoulder. A period of activity in the pitch coordinates follows the completion of the soft compressive phase. Yawing of the man and rifle was small compared with the pitching rotations. Yaw was minimized by the rigid stance of the man and his initial positioning of the rifle.

3.3 M79 and M203 Grenade Launcher Tests

The grenade launcher tests were performed on an experimental setup similar to the M16 experiment (see Figure 6). One difference was that fixed points on the man and the weapon were identified by circular pieces of reflective tape. The tape markings helped to simplify the data reduction on the Vanguard Motion Analyzer. Another modification in the grenade launcher tests was that the rotorary prism type camera was replaced with a shutter type camera, operating at a slower 500 frames/second, to improve the film resolution. The reduced film speed is sufficient to identify when the grenade exits the barrel and to obtain continuous time history data of all significant motions. Several film sequences were recorded with different size men who fired from both the shoulder and hip configurations. The data were reduced, on the Vanguard Motion Analyzer, by the Intertech Corporation in Iowa City, Iowa. In reduced form, the data provided pitch and yaw rotations of the weapon along with the pitch of the man and the weapon displacement.

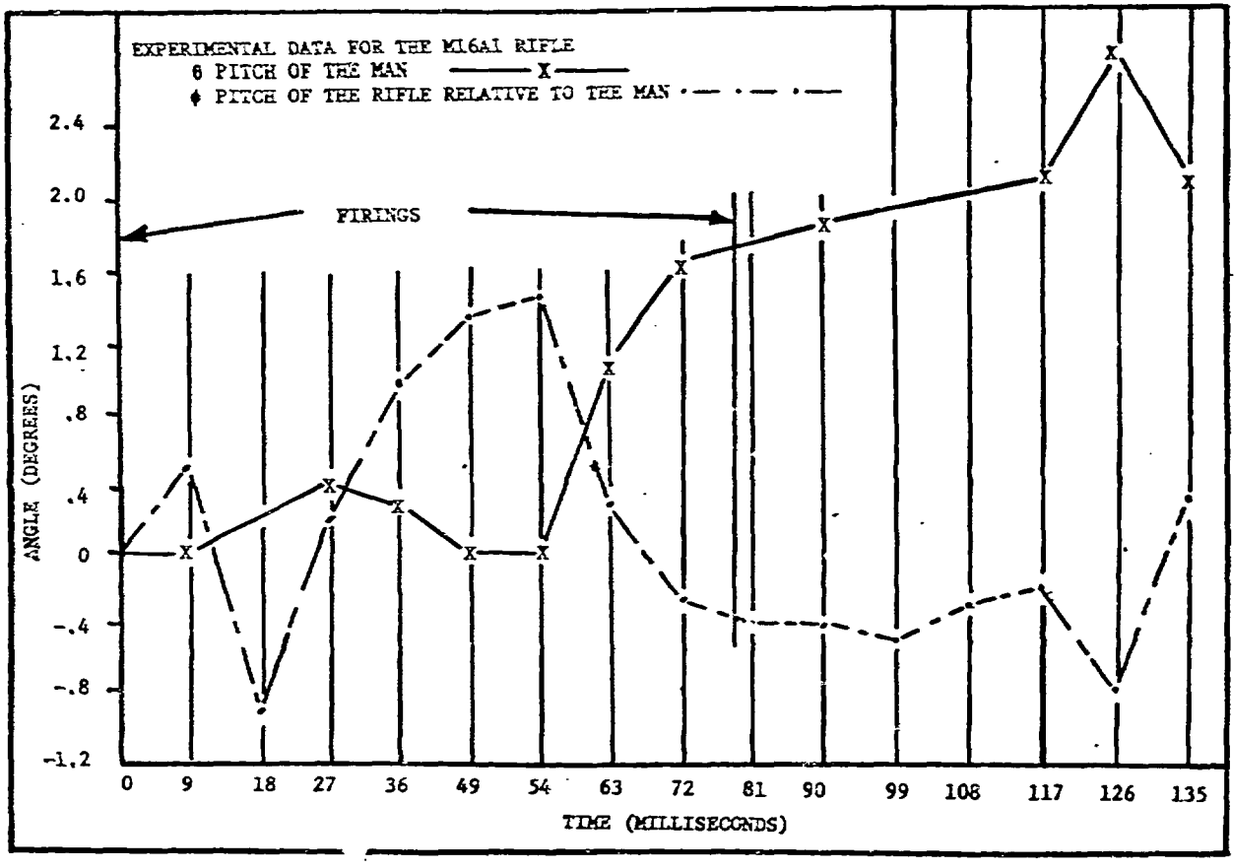


Figure 5 Experimental Data For The M16A1 Rifle

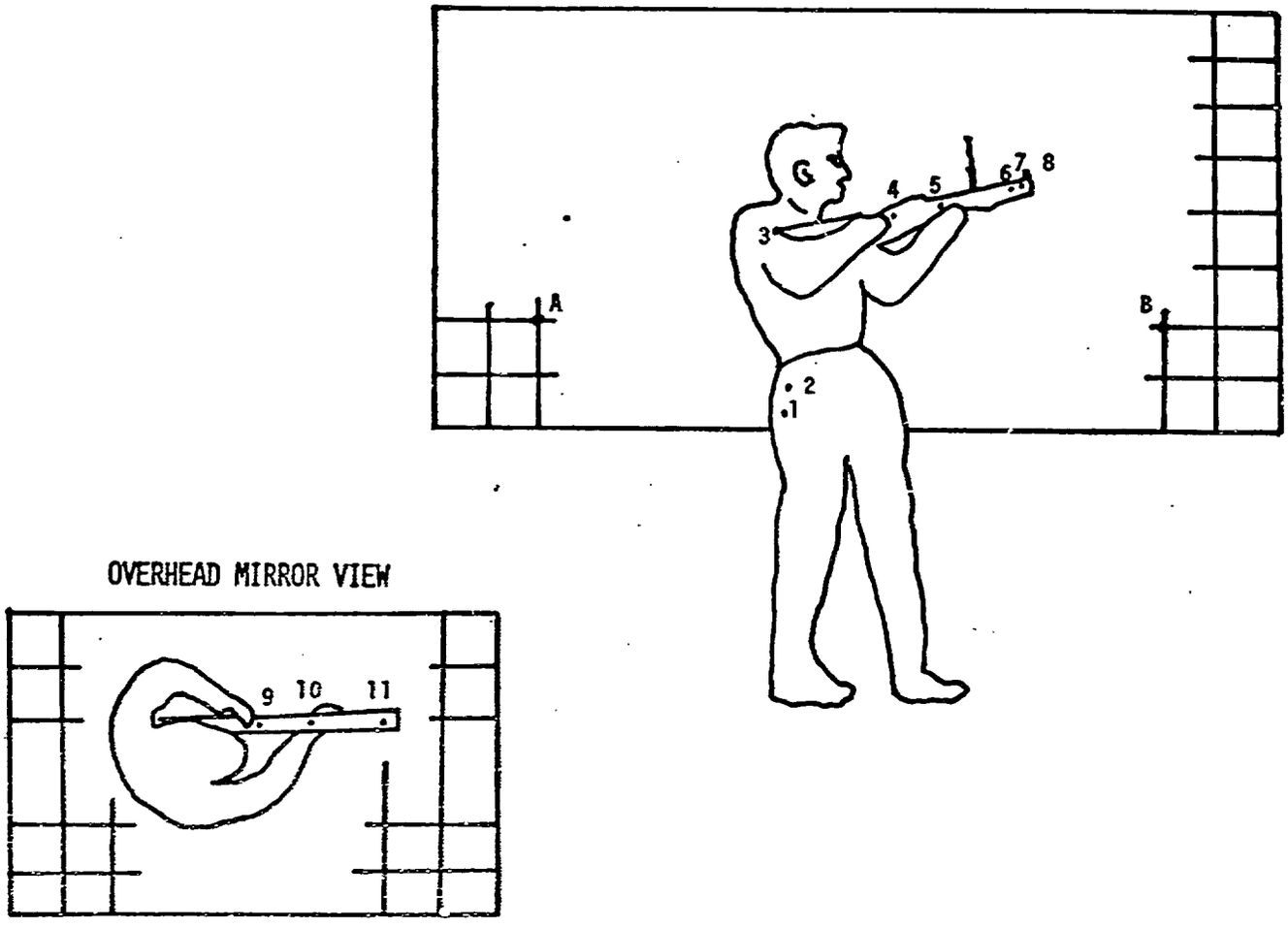


Figure 6 M79 and M203 Grenade Launcher Experiment

The M203 grenade launcher data were the most thoroughly studied data. The M203 grenade launcher is mounted below the barrel on the M16 rifle, thus causing this weapon to pitch downward rather than upward, as most other weapons do. Also the data for the shoulder support configuration reveal that the pitch rotation of the weapon varies somewhat with the size of the shooter. There is less pitch of the weapon for a heavier man than for a lighter man. This is expected because the heavier man has greater inertia. Firings from the hip produced less weapon pitch than firings from the shoulder. Again, this is expected since the constraints on the weapon in the hip firing position tend to limit the amount of weapon pitch rotation.

Plots of the total weapon rotation versus time have been generated for the test firings of the M79 and M203 grenade launchers. Several of the plots for the M203 grenade launcher tests appear in Figures 7 to 13. The test conditions for these film sequences are listed in Table 1. Each plot contains both the measured weapon pitch angle data points, connected by straight line segments, and a fourth order least square polynomial approximation to the data. The time interval begins at the commencement of firing and ends approximately when the man's active neuromuscular response starts to influence the motions of the system (i.e. at roughly 0.3 second). In each film sequence, the weapon pitches downward from the initial aim elevation angle to a maximum value of about seven degrees. The shapes of the plots vary somewhat both for different shooters and different aim elevations. The plots in Figures 11 and 12 reveal that different test results were obtained even for tests repeated under identical conditions. This may be due partly because of errors in data reduction and because the shooter does not prepare himself in exactly the same way for each shot.

TABLE 1 M203 GRENADE LAUNCHER TEST CONDITIONS

FILM	SEQUENCE	SHOOTER WEIGHT (LB)	ELEVATION ANGLE (DEG)
8	2	140	5°
8	4	140	10°
8	6	140	20°
9	3	165	5°
9	5	165	10°
9	6	165	10°
9	7	165	20° - 25°

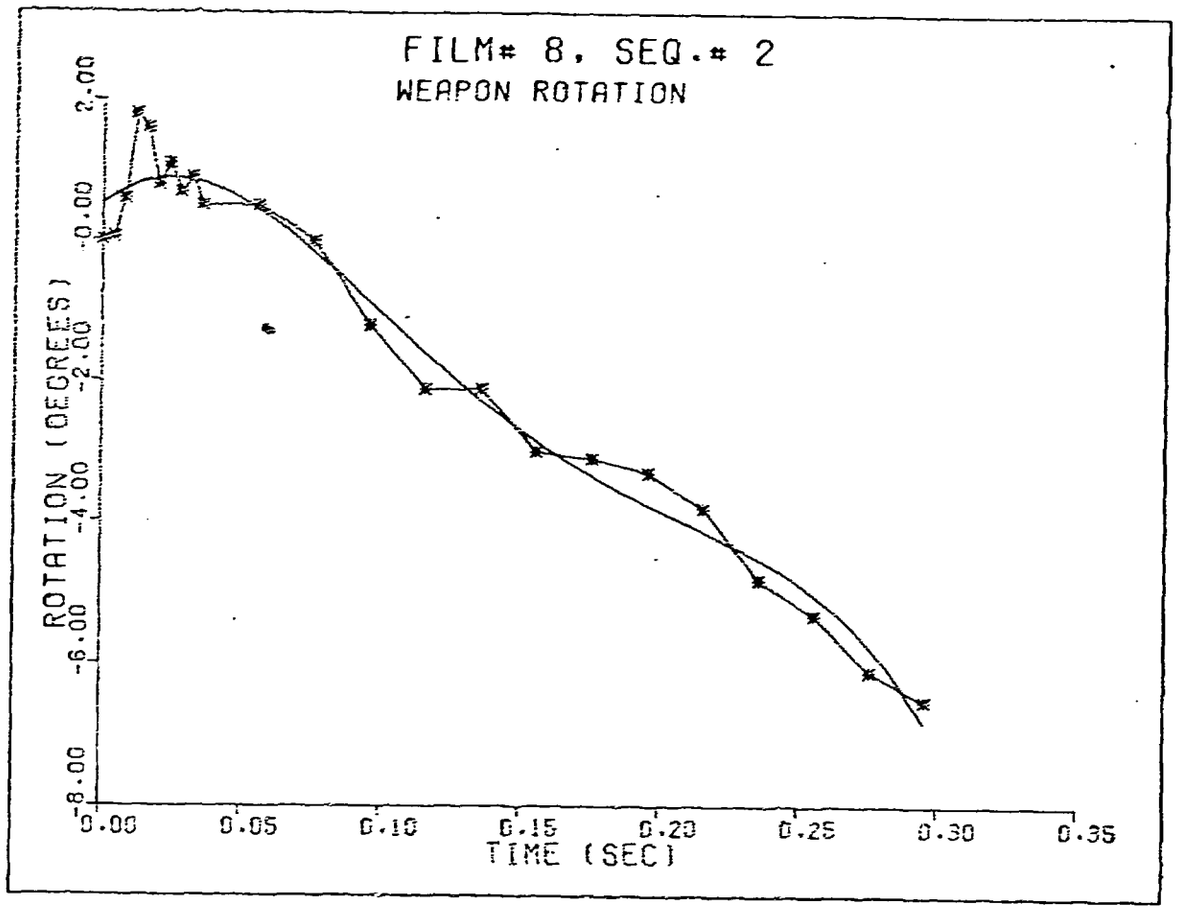


Figure 7 Film# 8, Seq.# 2 - Weapon Rotation

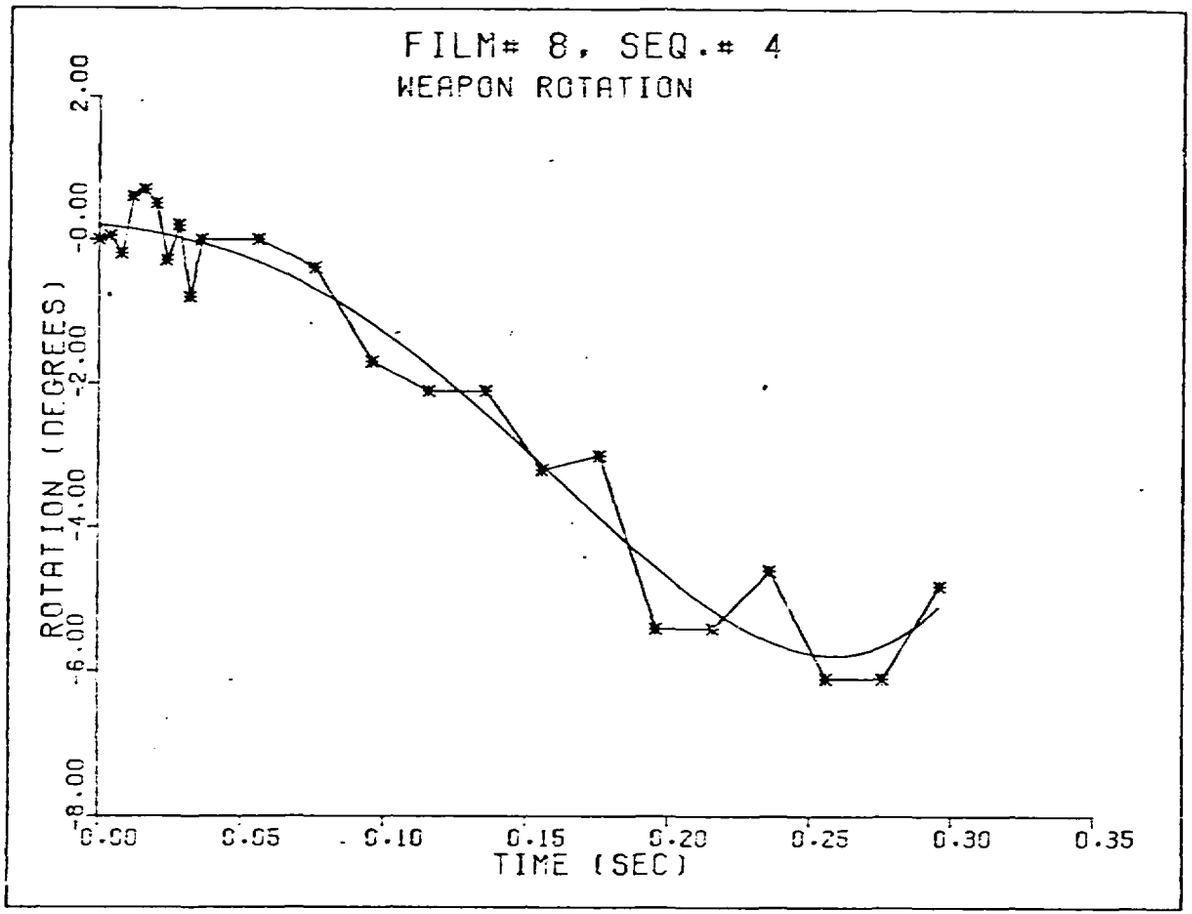


Figure 8 Film# 8, Seq.# 4 - Weapon Rotation

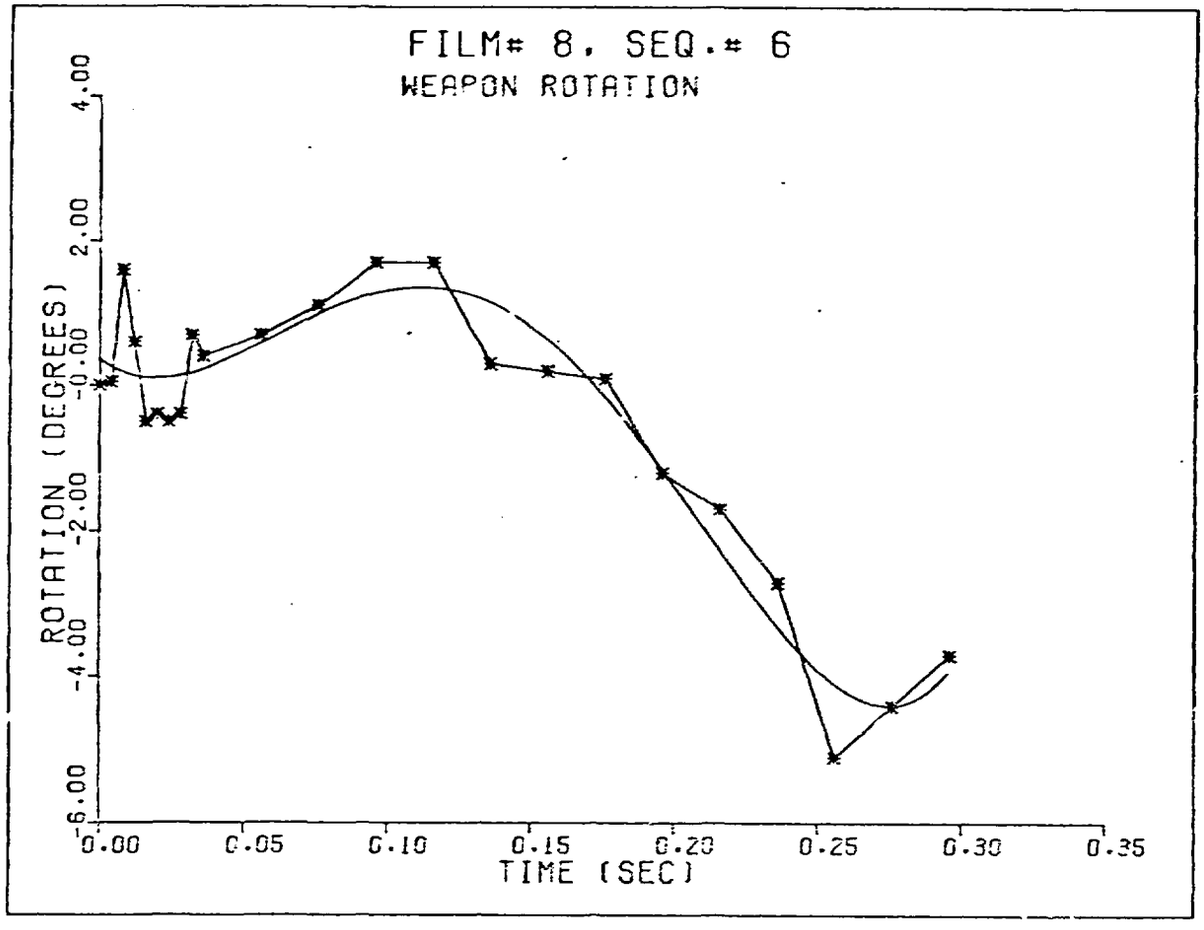


Figure 9 Film# 8, Seq.# 6 - Weapon Rotation

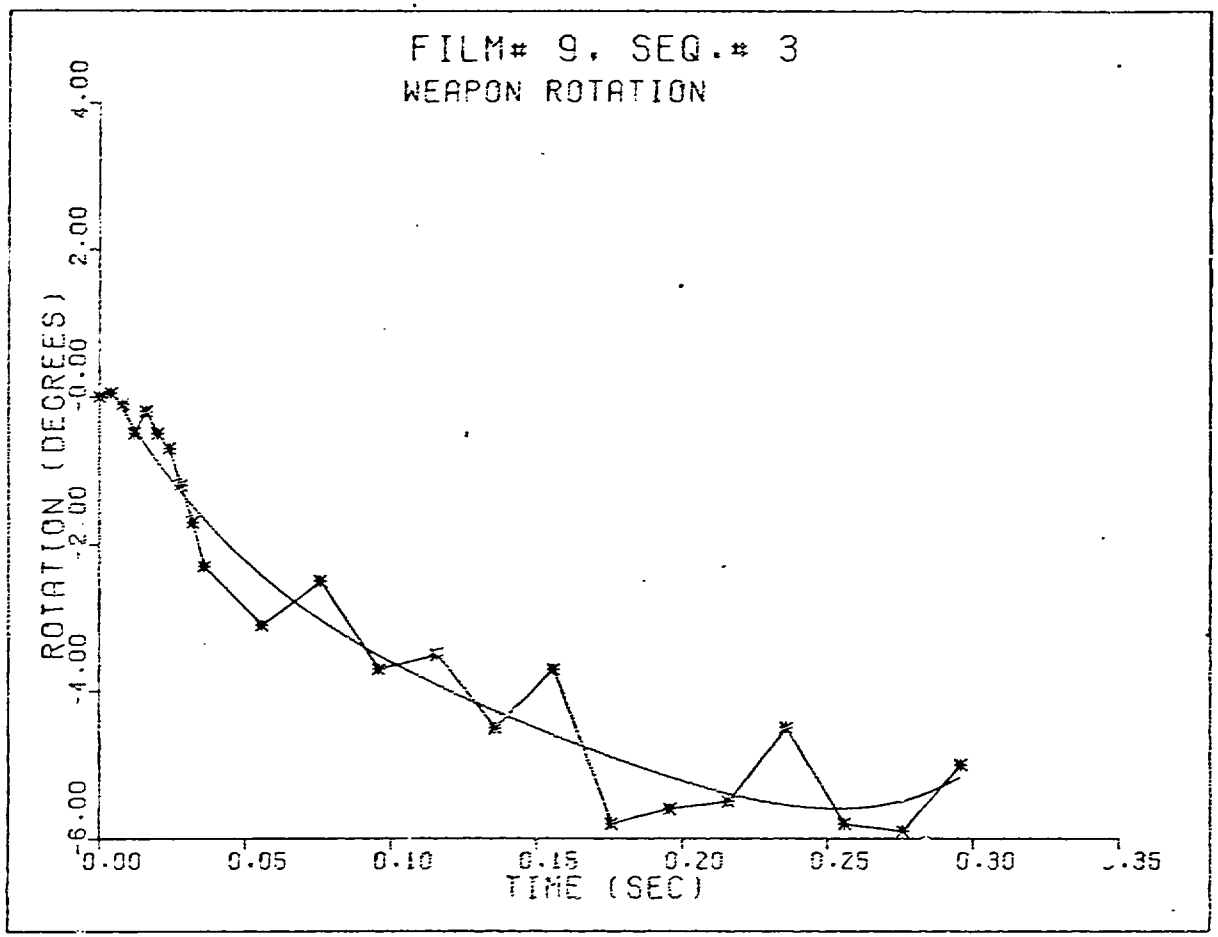


Figure 10 Film# 9, Seq.# 3 - Weapon Rotation

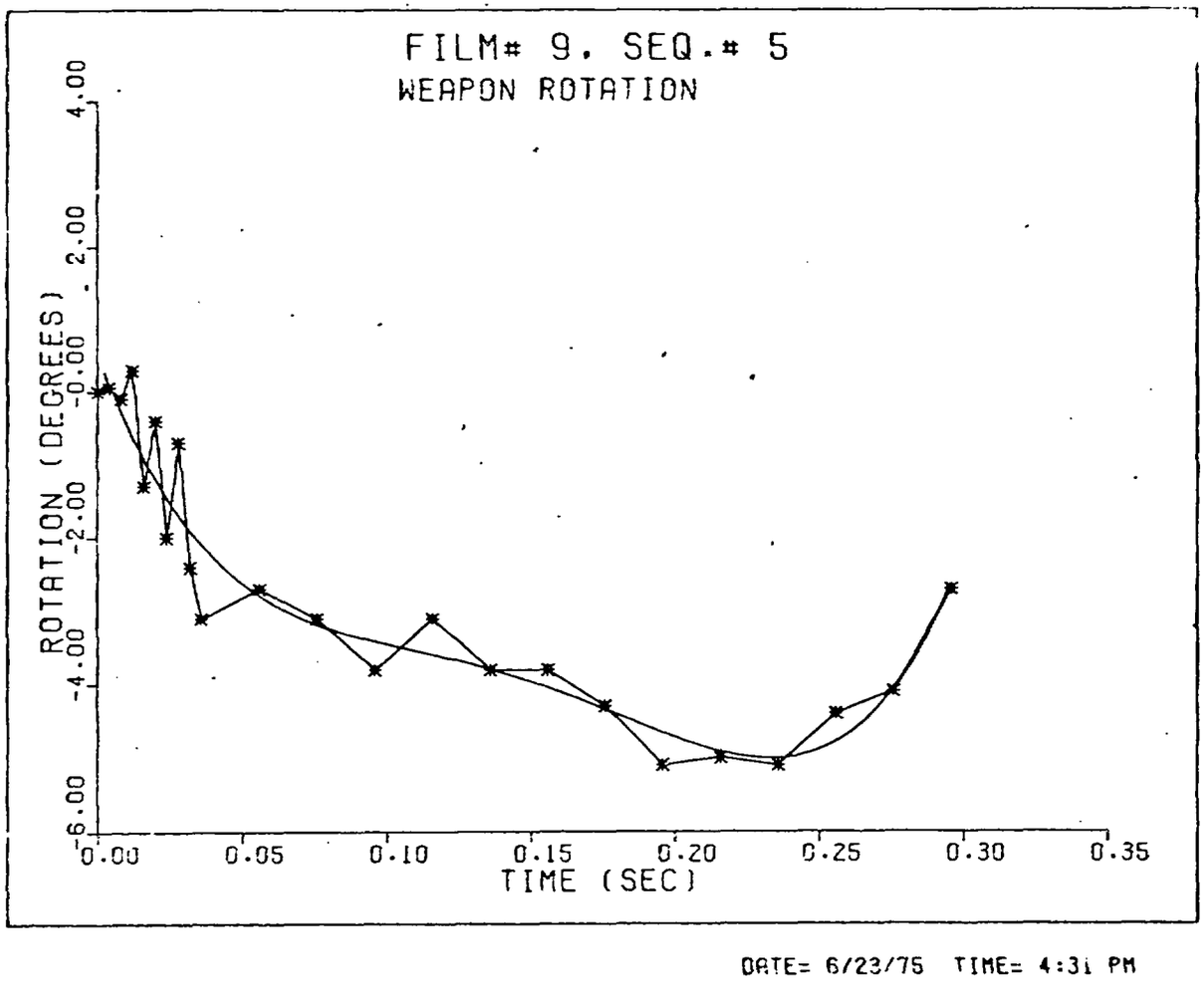


Figure 11 Film# 9, Seq.# 5 - Weapon Rotation

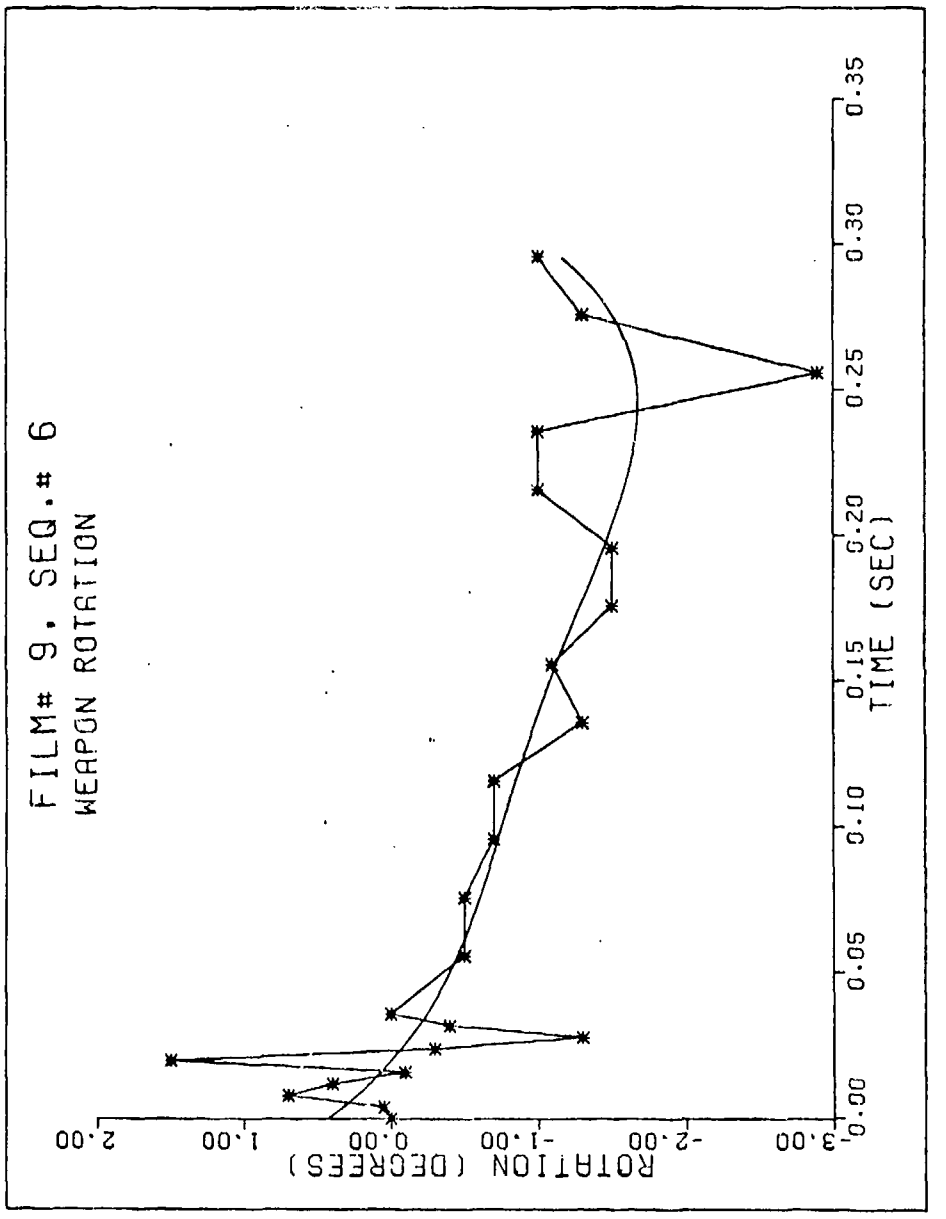


Figure 12 Film# 9, Seq.# 6 - Weapon Rotation

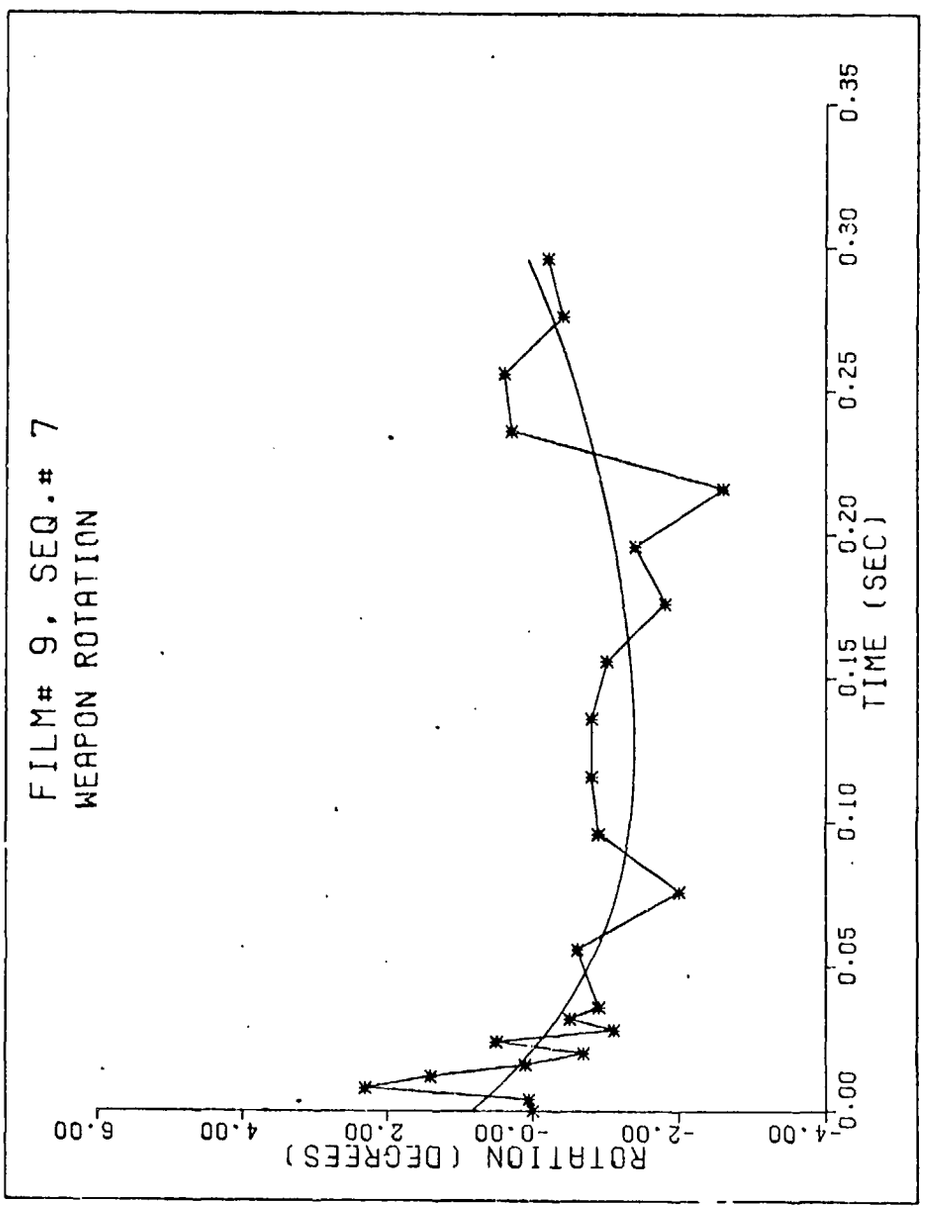


Figure 13 Film# 9, Seq.# 7 - Weapon Rotation

3.4 Measurement of the Shoulder Reaction Force

Additional M16 data were obtained at the Keith L. Ware Simulation Laboratory. The experimental setup is shown in Figure 14. A shoulder fixture (see Figure 15) containing load cells and an accelerometer was used to measure the weapon recoil force transmitted to the man's shoulder. The force reaction at the shoulder is obtained by subtracting the inertial force of the fixture from the measured load cell force (see Figure 16). The linear potentiometers measured the vertical displacement of the weapon at two points on the rifle. Referring to Figure 17, the weapon pitch rotation, θ, is then calculated from the difference between the two vertical displacements.

The most significant conclusion that was drawn from the Ware Laboratory data is that the shoulder reaction force is of very short duration (i.e. impulsive) compared with the rifle firing cycle time. The maximum forces ranged from 20 to 180 pounds. Peak recoil forces tended to be greater for the larger riflemen than for the smaller riflemen. Also, the peak values tended to decrease significantly from round to round. The ratios of peak values for successive shots varied considerably among the tests. Ratios for successive peak recoil forces in the neighborhood of 0.85, however, were common.

The data gathered from these experiments provided insight for constructing the simulation model and provided a means for estimating some of the unknown parameters in the model (i.e. stiffness and damping constants). The time history data obtained by high speed photography is more reliable than the data measured by linear potentiometers, because the photographic technique does not have any direct influence on the experiment. On the other hand, the use of high speed photography to measure motions has the disadvantage of requiring a large amount of time and effort in data reduction.

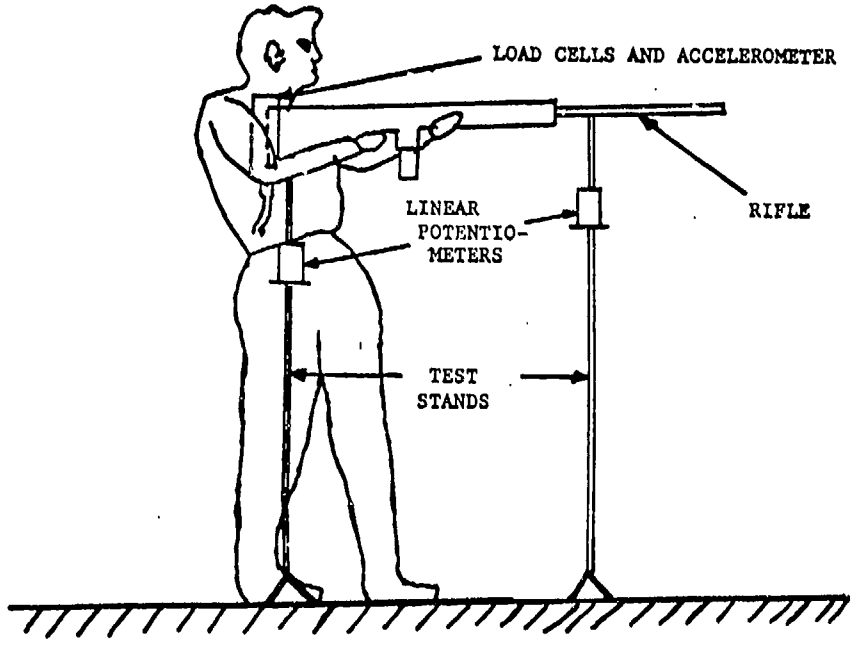


Figure 14 Measurement of Recoil Force and Weapon Pitch

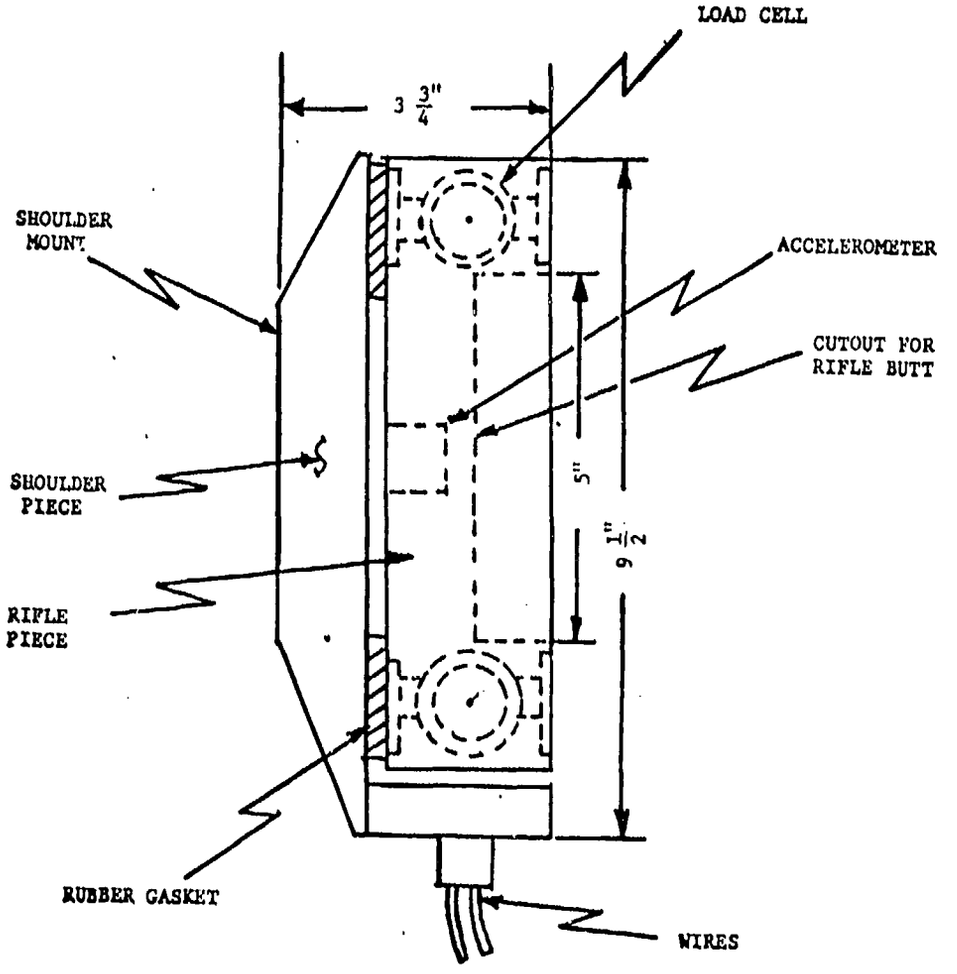
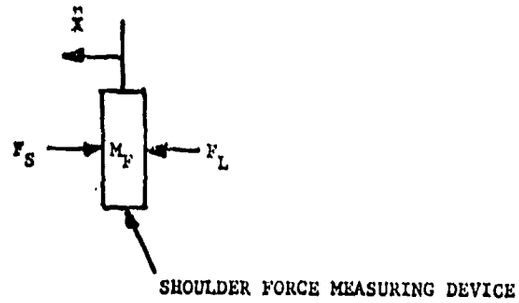


Figure 15 Shoulder Force Measuring Device



M_F - MASS OF FIXTURE

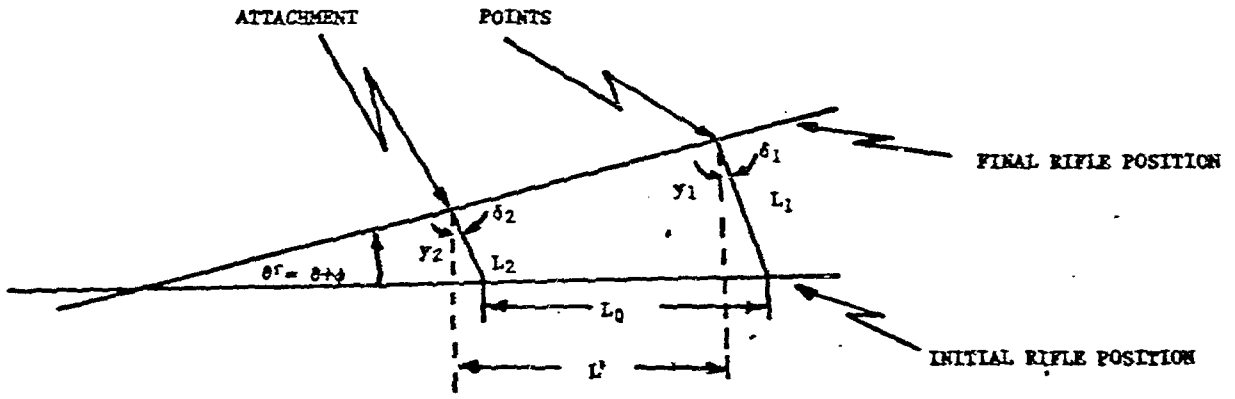
F_L - MEASURED FORCE (LOAD WITH CELLS)

$\ddot{X}$ - MEASURED ACCELERATION

F_S - COMPUTED FORCE EXERTED ON THE SHOULDER

$$F_S = F_L - M_F\ddot{X}$$

Figure 16 Computation of Transmitted Recoil Force



θ', δ_1, δ_2 - SMALL ANGLES (less than 10°)

$L' = L_0 \cos \theta' \approx L_0$

$Y_1 = L_1 \cos \delta_1 \approx L_1$

$Y_2 = L_2 \cos \delta_2 \approx L_2$

$$\tan \theta' = \frac{Y_1 - Y_2}{L'} \approx \frac{L_1 - L_2}{L_0}$$

Figure 17 Computation of Weapon Pitch Rotation θ

4.0 BIODYNAMICAL SENSITIVITY ANALYSIS

4.1 Introduction

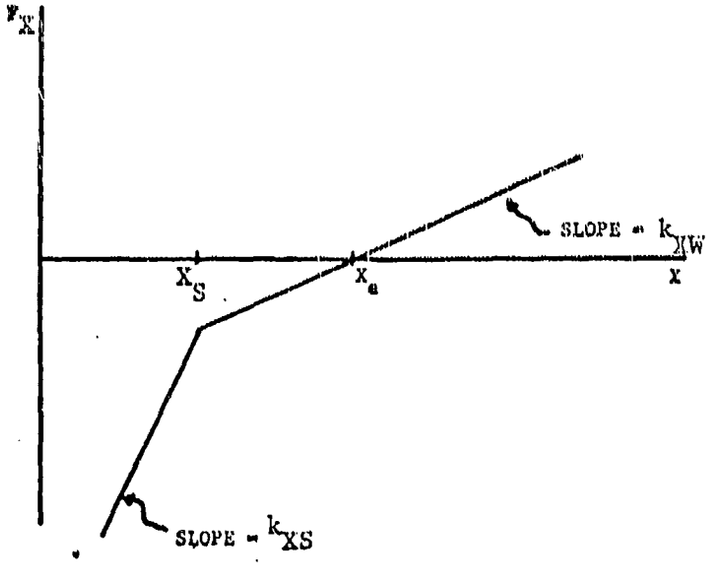
A sensitivity analysis was performed to determine which biomechanical parameters of the man-weapon system are critical to the design of a universal small arms mount. In this analysis, both the peak force at the shoulder and the absolute weapon pitch rotation after 0.3 seconds were taken as the primary output variables. The sensitivities of these output variables to systematic variations in each of the system parameters were obtained by numerical and analog computer solutions of the biodynamical equations of motion.

Both single and burst fire modes were analyzed. The driving force F(t) in equations 2-7 represents what is commonly called the breech force. The important feature of this function is the short duration (about one millisecond) compared with the total cycle time (approximately 80 milliseconds). For the digital computer simulation, the function F(t) is approximated by Dirac's delta function -- a symbolic function commonly used in mathematical physics that has finite area but is zero valued at all but one point on the time axis. Typical values for the impulse are 1.2 lb-sec for the M16 Rifle and 2.5 lb-sec for the M79 and M203 grenade launchers.

4.2 Factors That Affect Weapon Translation

The weapon shoulder support is represented in the model by a linear dashpot and a piecewise linear combination of spring elements, which results in variable shoulder stiffness. Thus the shoulder spring model (illustrated in Figure 18) represents the effects of both the soft compression of skin tissue and the stretching of the muscles and tendons that support the shoulder. The shoulder reaction force, F_s, is an explicit function of coordinate x, but since the man-weapon system is dynamically coupled, the shoulder reaction force also is influenced, to a lesser extent, by both the pitching coordinates θ and ϕ.

Because the shooter usually holds the weapon compressed firmly against his shoulder before firing and, in addition, because the impulsive breech force compress the weapon against the shoulder, the shoulder support intuitively is expected to be in a state of strong compression (i.e. $x < x_s$) for most of the firing cycle. This hypothesis was substantiated by numerous computer simulations of the man-weapon interaction problem and also by observations of experimental load cell data of the shoulder reaction force.



$$F_X = \begin{cases} k_{XW}(X_S - X_e) + k_{XS}(X - X_S) & \text{FOR } X < X_S \\ (X - X_e) & \text{FOR } X \geq X_S \end{cases}$$

$$F_S = F_X + C_X \dot{X}$$

WHERE

X_e = DISTANCE AT WHICH THE WEAPON AND SHOULDER ARE IN STATIC EQUILIBRIUM

X_S = DISTANCE AT WHICH THE SPRING STIFFNESS CHANGES

k_{XW} = WEAK SHOULDER SPRING CONSTANT

k_{XS} = STRONG SHOULDER SPRING CONSTANT

C_X = SHOULDER DAMPING CONSTANT

F_X = SHOULDER SPRING FORCE

F_S = TRANSMITTED SHOULDER MOUNT FORCE

Figure 18 Shoulder Spring-Dashpot Model

A flexible shoulder model, consisting of a single linear shoulder spring and a dashpot, therefore provides an adequate representation of the shoulder impedance to weapon recoil loading.

Experimental results also indicate that the shoulder reaction force, like the breech force, is both impulsive and nearly periodic; and, in addition, the magnitude of the peak value of the shoulder force tends to decrease for successive shots in a burst. If the model damping coefficients are near critical, then the shoulder reaction force is spread over a longer portion of the firing cycle, and, furthermore, the peak values of the shoulder reaction force become sensitive predominantly to the damping term rather than the spring stiffness term. In order to duplicate the impulsive and the periodic character of the shoulder force, a damping factor between 10 and 30 percent of critical damping is required (i.e. $c_x = 2\xi_x\sqrt{k_x m_r}$ where $0.1 \leq \xi_x \leq 0.3$). A spring stiffness of 250 lb/in yields peak values for the reaction force of about 100 pounds. Using these values for the shoulder spring and dashpot parameters, the model generated shoulder force closely approximates the characteristics of the experimentally measured recoil force.

Results of the computer analysis reveal that only four parameters have any significant effect on the duration or the peak values of the recoil force, namely: the weapon impulse, I_r; the combined mass of the weapon and the arms, m_r; the spring stiffness, k_x; and the damping constant, c_x. Analog computer simulations were made using inertial parameters associated with three different shooters (small, medium, and large) and these results are shown in Figures 19, 20, and 21 respectively. The shoulder damping constant, ranging from 10 to 30 percent of critical damping, was adjusted to give average peak recoil forces that correspond to measured results from experimental firings. As expected, the model results show a decrease in the weapon rotation as the size of the man increases. Only slight variations in the recoil force occur among the three cases.

4.3 Factors That Affect Weapon Pitch Rotation

Certain system parameters were found by computer analysis to have a significant effect on the absolute weapon pitch rotation, while other parameters were found to have very little effect on weapon rotation at all. Table 2 contains a list of those parameters that have very little effect on the rifle rotation. The torso mass, moment of inertia, and the torsional

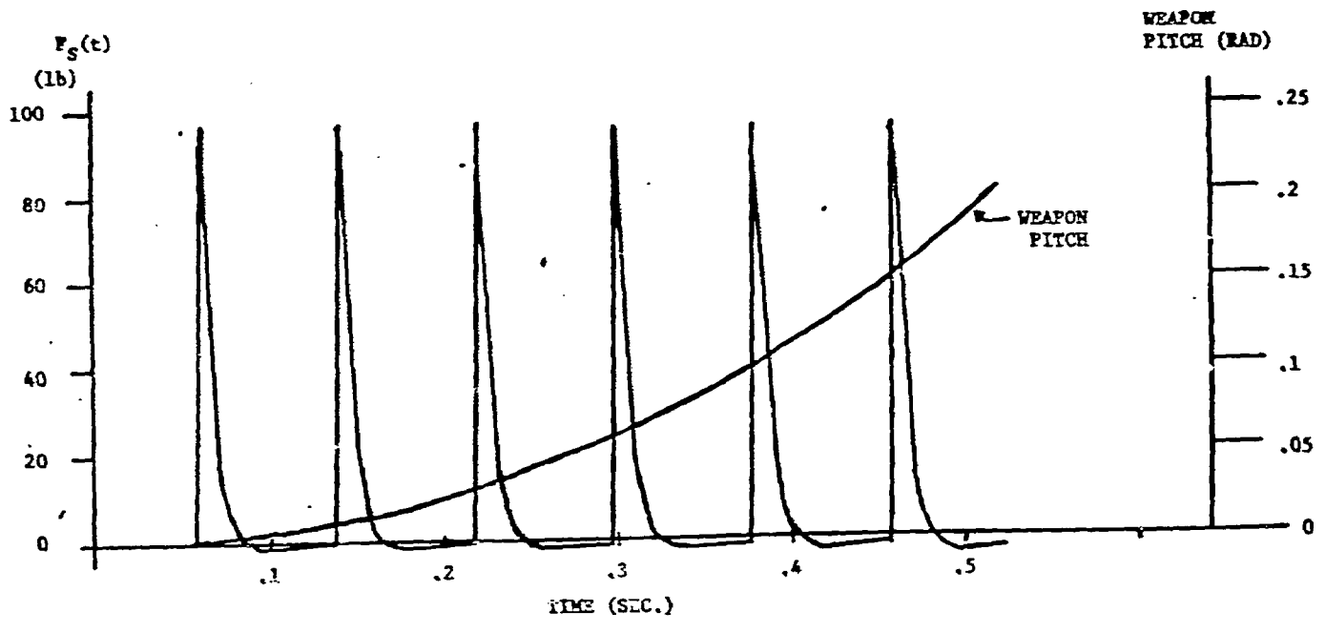


Figure 19 Analog Computer Simulation - Small Man
Gunner: Behling (145 lb, 5'6")

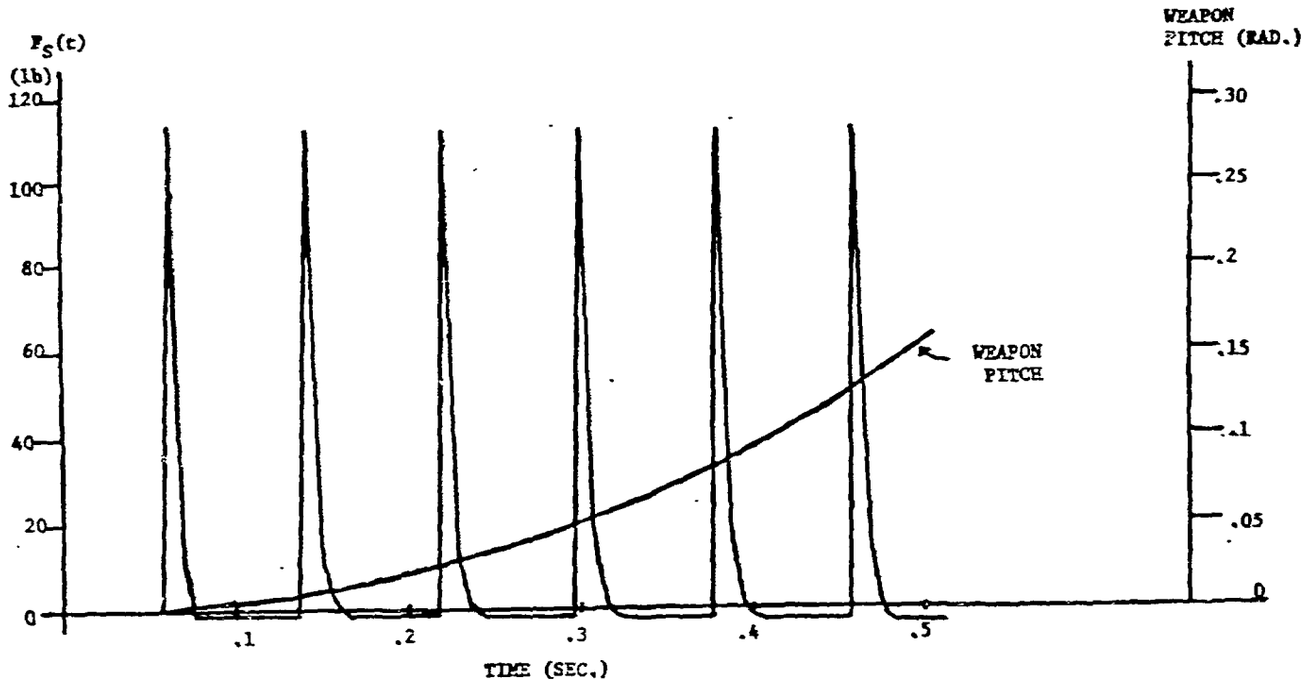


Figure 20 Analog Computer Simulation - Medium Man

Gunner: Rekemeyer (165 lb, 6'1½")

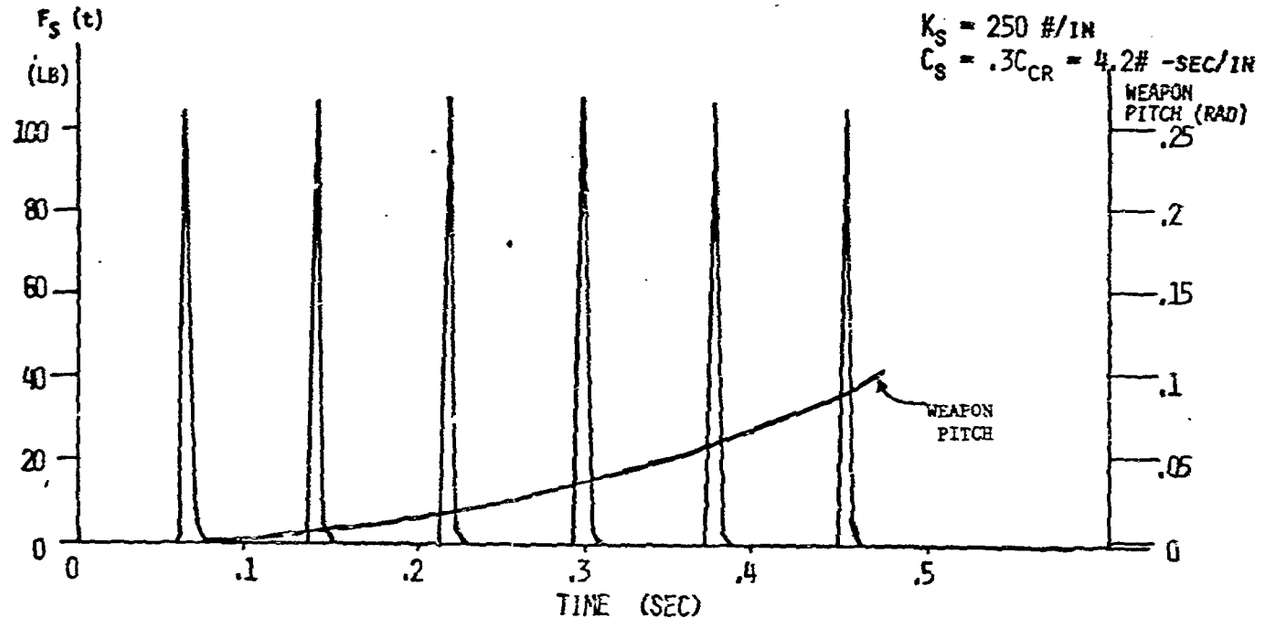


Figure 21 Analog Computer Simulation - Large Man

Gunner: Boggs (228 lb, 5'11")

TABLE 2 PARAMETERS THAT DO NOT AFFECT WEAPON ROTATION

PARAMETER	DECREASED VALUE	RIFLE ROTATION	BEST APPROX.	RIFLE ROTATION	INCREASED VALUE	RIFLE ROTATION
k_0 (in-lb)	2.	3.84°	20.	3.84°	200.	3.83°
$\xi_\theta = \frac{c_0}{c_{0\ crit}}$	0.	3.84°	0.5	3.84°	1.	3.83°
I_{cm}(lb-in-sec^2)	12.	3.87°	26.	3.84°	50.	3.83°
M(lb-sec^2/in)	0.15	3.77°	0.3	3.84°	0.45	3.84°
L (in)	10.	4.07°	21.	3.84°	30.	3.60°
r_{cm}(in)	5.	3.73	13.	3.84°	25.	3.90°

stiffness and damping coefficients for the θ coordinate do not affect the rifle rotation to any appreciable amount (a few hundreths of a degree at most). The weapon rotation is rather insensitive also to the length of the torso and the distance from the torso pivot point to the center of mass of the torso. These results provide some insight for designing a mount. They suggest that a representation of the torso is not really necessary for the design, in order to simulate the external dynamical motions of the weapon. In the model, the motion of the torso mass can be reduced to near zero by simply assigning a very large mass and moment of inertia. For example, by increasing the mass and moment of inertia by a factor of one thousand, the rotation of the weapon increases from a value of 3.84 degrees to a value of 4.06 degrees (see Figure 22). Although the torso rotation is reduced from 7.0 degrees in the nominal case to zero by increasing the mass and moment of inertia, the weapon rotation changes only by 0.2 degrees; consequently, the torso mass has very little effect on the total rotation of the weapon.

Figure 23 contains a plot of the weapon pitch versus the weapon impulse after 0.3 seconds have elapsed. Note the linear relationship between these two parameters and the significant effect the impulse has on the weapon rotation. The point marked "best approximation" on this and on other figures corresponds to the set of parameters that best represents the

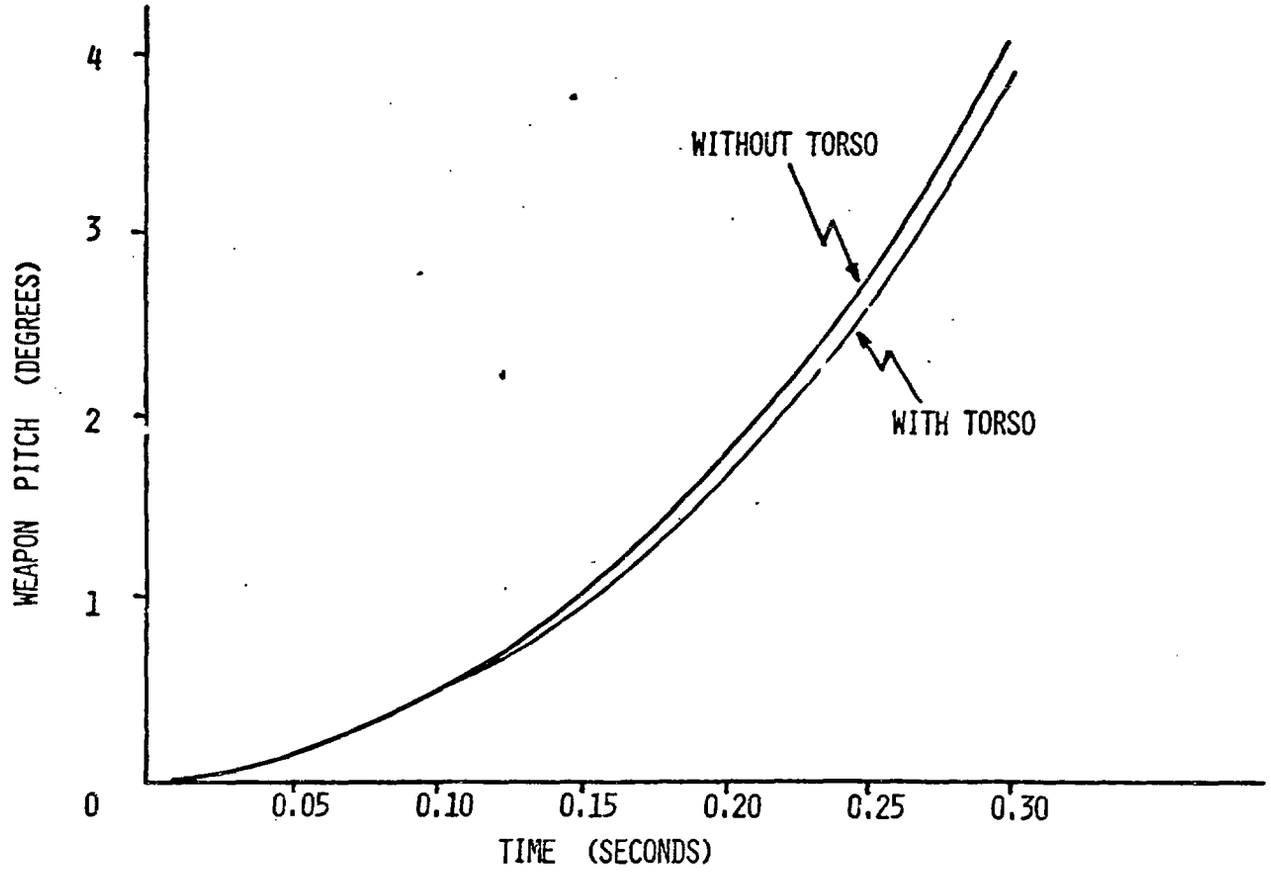


Figure 22 Sensitivity of Weapon Rotation to Torso Inertia

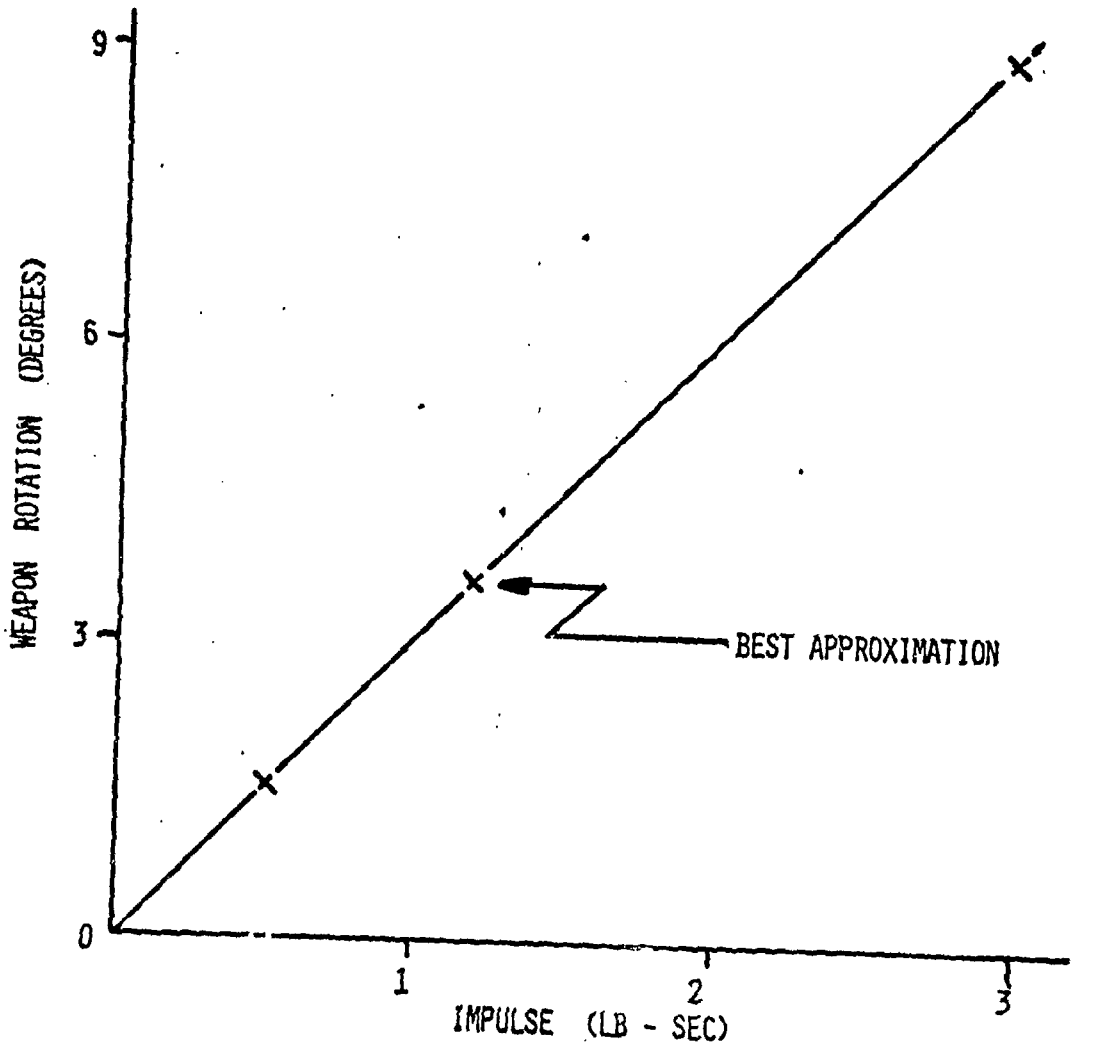


Figure 23 Weapon Pitch Rotation vs Impulse

dynamical characteristics of the 165 pound (medium sized) man. Figure 24 contains a plot of the weapon rotation at 0.3 seconds and at 0.15 seconds versus the firing frequency. Again there is a linear relationship between the weapon rotation and the firing frequency. Also the important conclusion can be made that, for a fixed number of rounds in a burst, increasing the firing frequency decreases the rifle rotation and hence lessens the weapon dispersion. Figure 25 shows the effect of the eccentricity of the breech force, δ, on the weapon rotation. This parameter represents the distance between the centerline of the barrel and the center of rotation of the weapon at the shoulder. The weapon pitch rotation varies linearly with δ. When δ is zero no torque is produced about the shoulder pivot point; nevertheless, 1.5 degrees of rotation occur after 0.3 seconds, which is due mainly to the rearward pitching of the torso. As the torso pitches rearward, the rifle pitch angle, ϕ, becomes negative, since the total system angular momentum is conserved after the application of the impulsive breech force. By appropriately positioning the stock with respect to the barrel centerline, the rotation of the weapon can be virtually reduced to zero.

Parameters that have a moderate influence on the rotation of the weapon are the inertial, stiffness and damping parameters associated with the man's arms and the weapon, namely, I_r, m_r, k_ϕ, c_ϕ, k_x, and c_x. In Figure 26 the rifle rotation is plotted as a function of $1/\sqrt{I_r}$. A straight line approximately fits the points. Similarly, a plot of the rifle rotation as a function of $\sqrt{k_\phi}$, (see Figure 27), results in a linear relationship. Thus the total rifle rotation varies linearly with both $1/\sqrt{I_r}$ and $\sqrt{k_\phi}$. In Table 3 the rifle rotation at 0.3 seconds is listed for a range of values of the parameters m_r, c_ϕ, c_x, and k_x.

Figures 28, 29, 30, and 31 contain the output of a digital computer simulation. The case represented is that of a 165 pound man firing the M16 rifle. The total rotation of the weapon, shown in Figure 28 is the sum of the torso rotation, θ, and the weapon rotation relative to the torso, ϕ. A characteristic feature of the simulation is the jump in acceleration in each of the coordinates after each shot, due to the impulsive breech force.

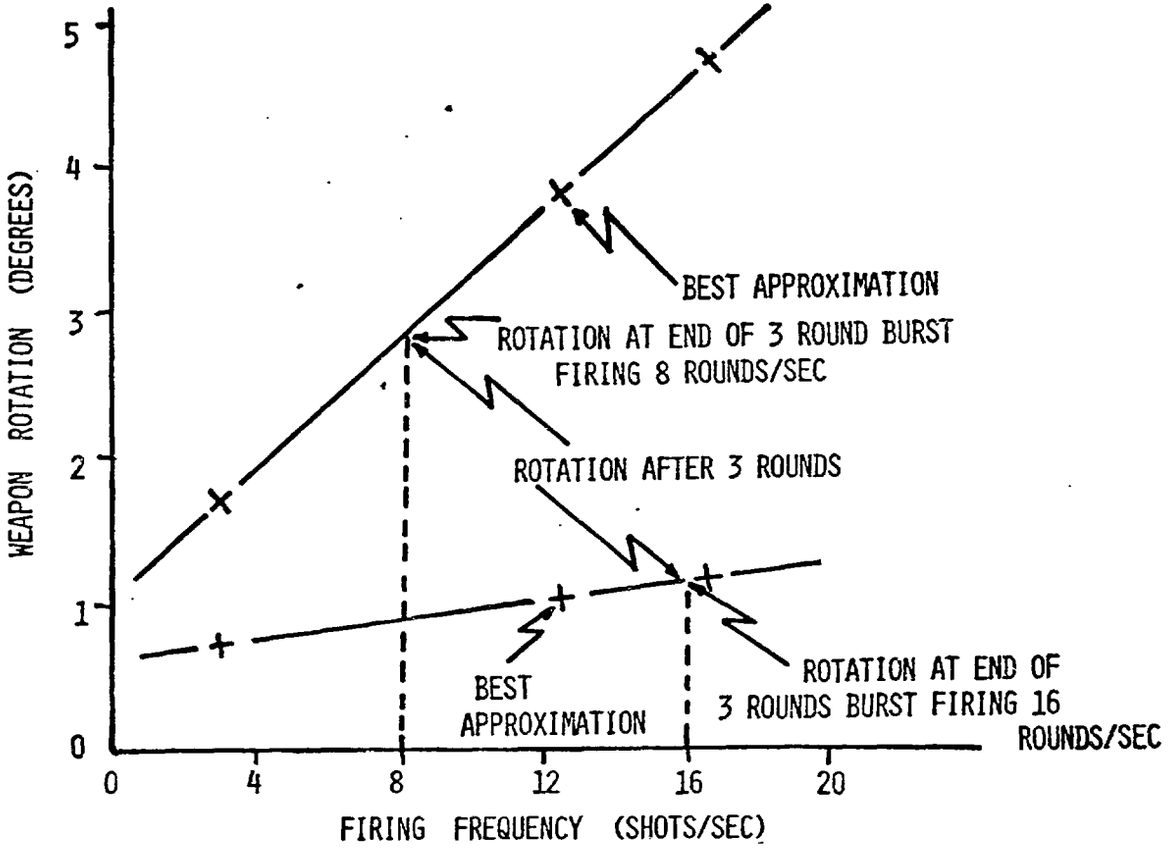


Figure 24 Weapon Pitch Rotation vs Firing Frequency

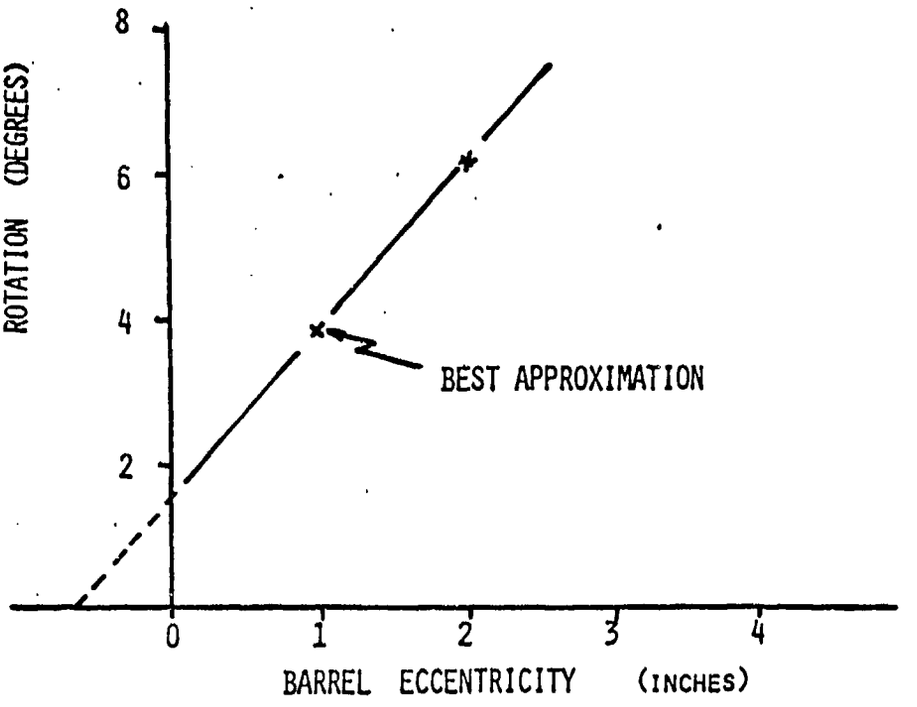


Figure 25 Weapon Pitch Rotation vs Barrel Eccentricity

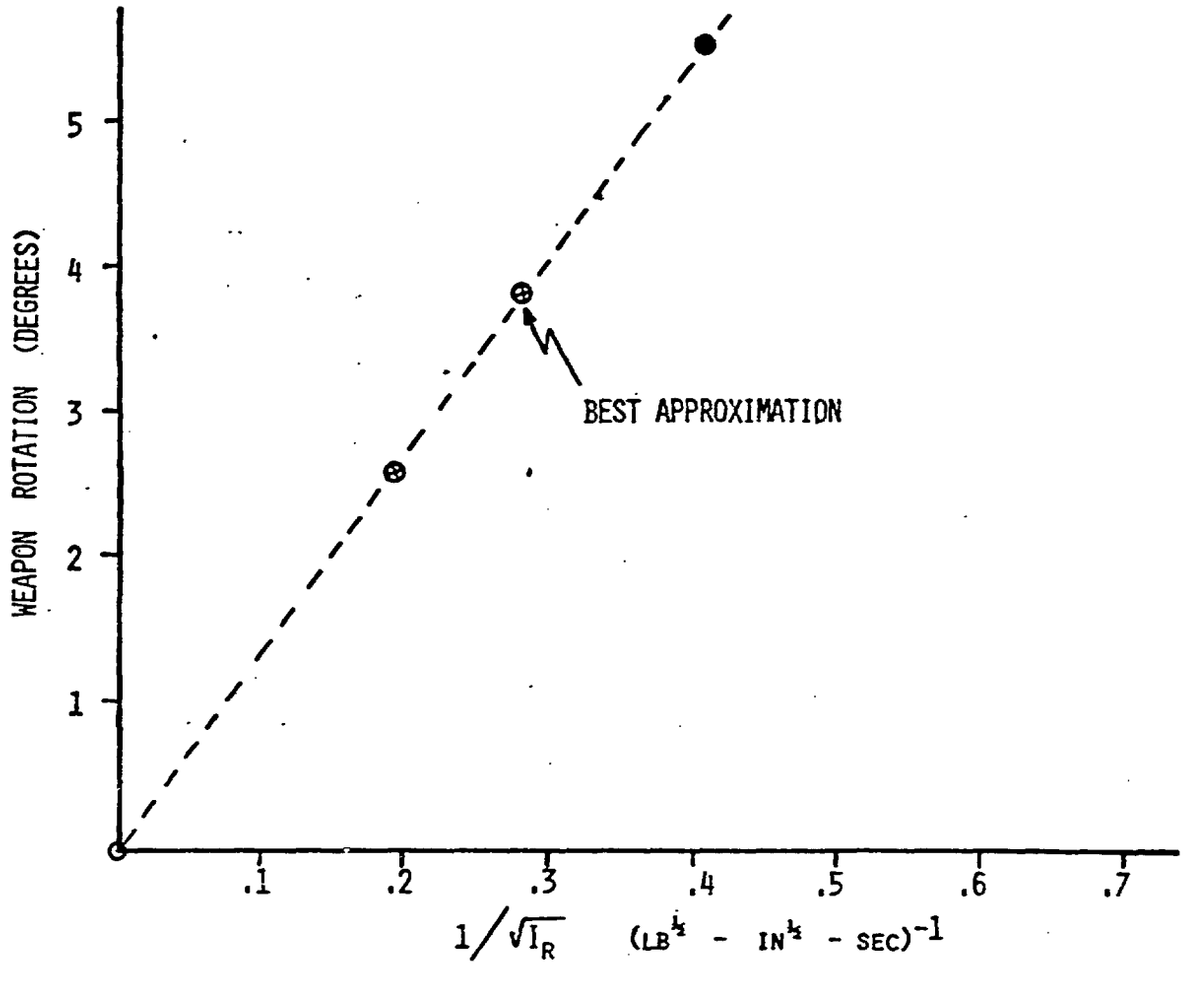


Figure 26 Weapon Pitch Rotation vs $1/\sqrt{I_r}$

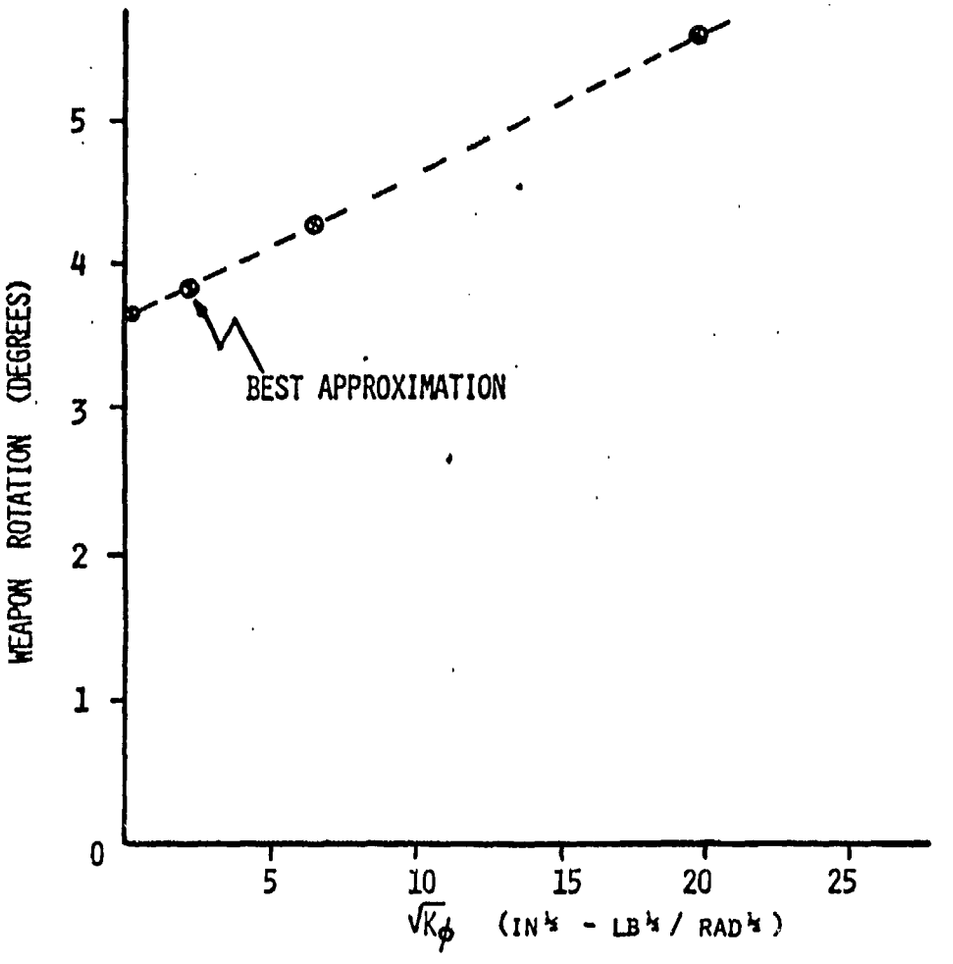


Figure 27 Weapon Pitch Rotation vs $\sqrt{K_\phi}$

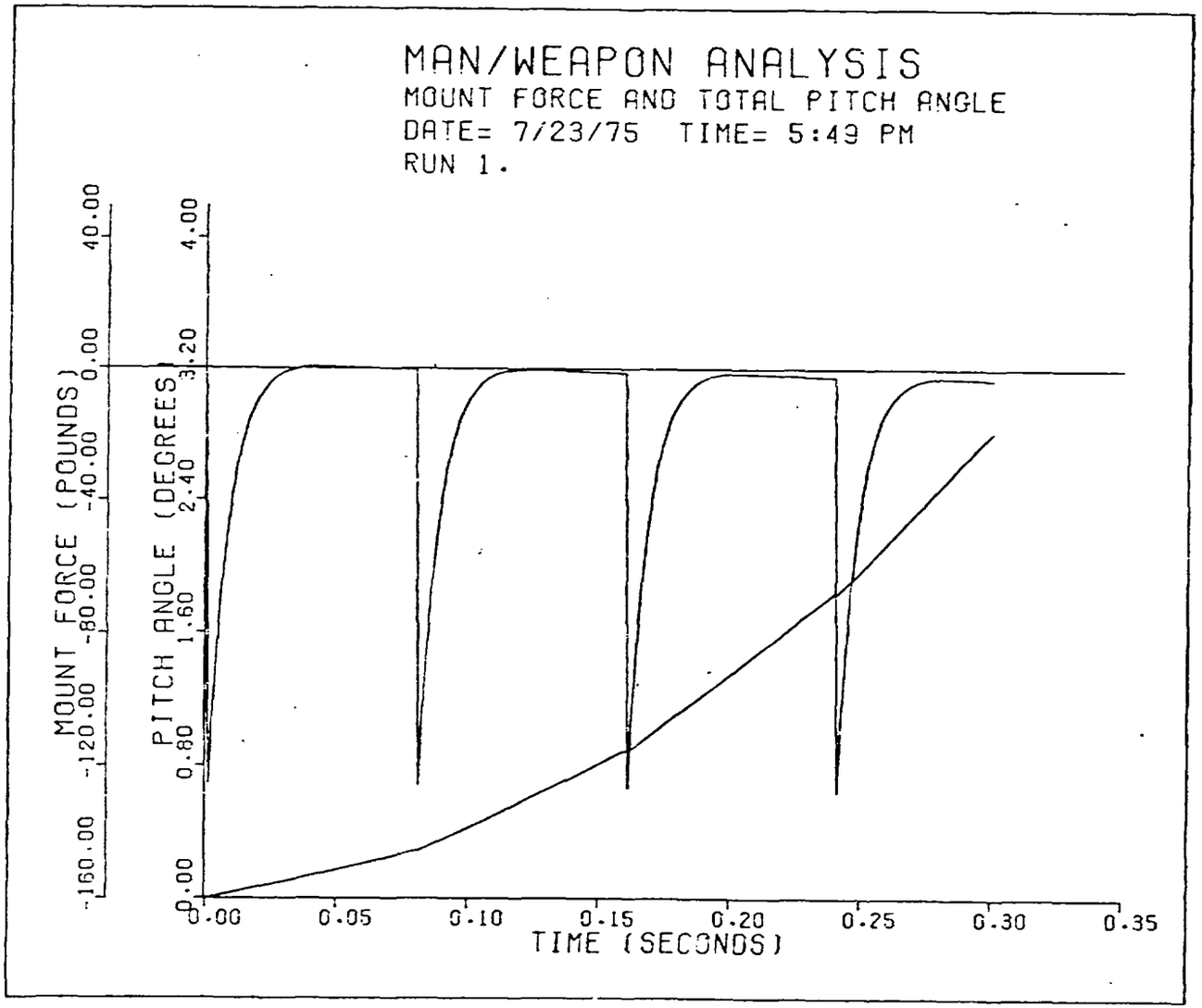


Figure 28 Man/Weapon Analysis- Mount Force and Total Pitch Angle

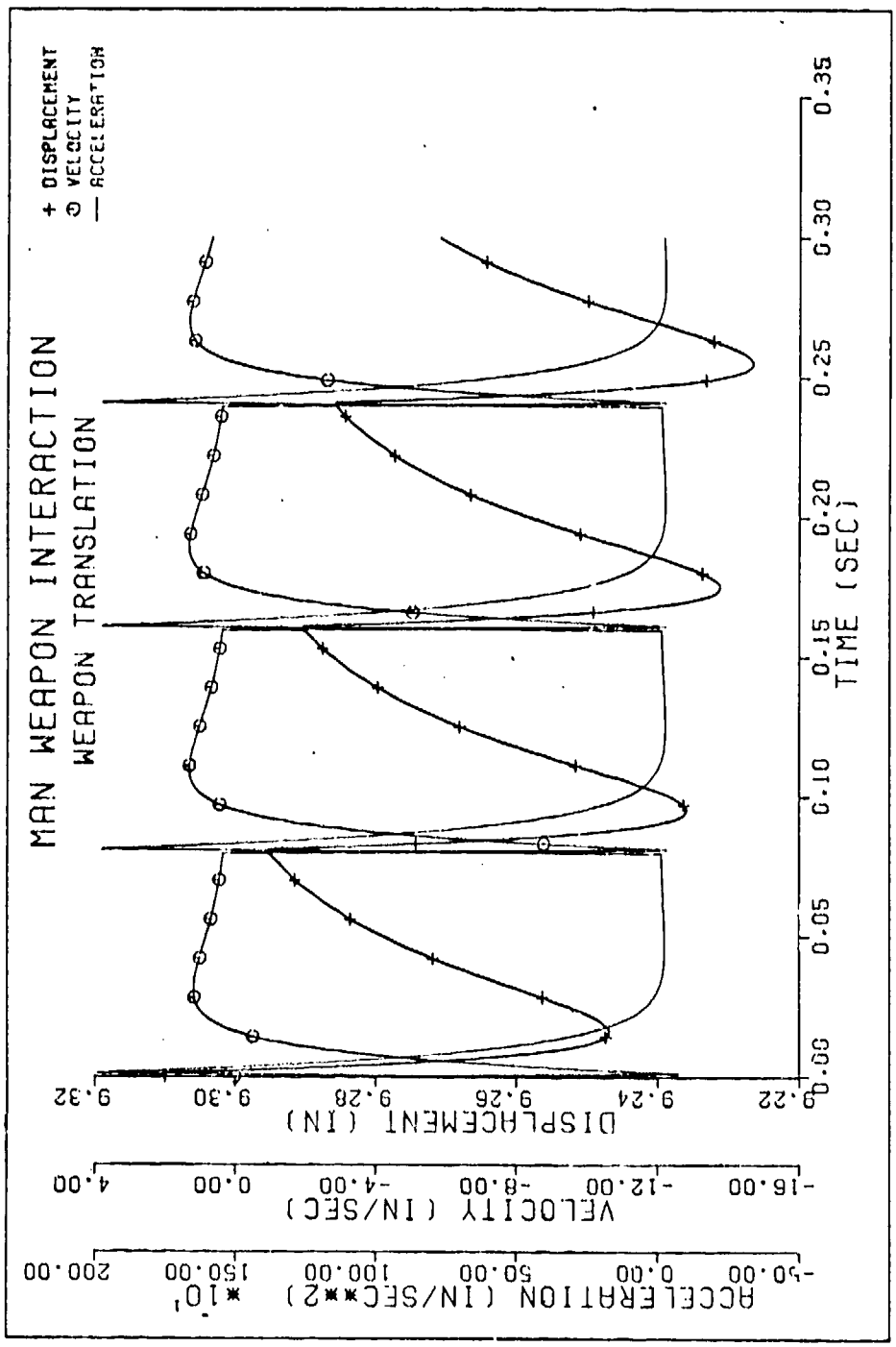


Figure 29 Man Weapon Interaction- Weapon Translation

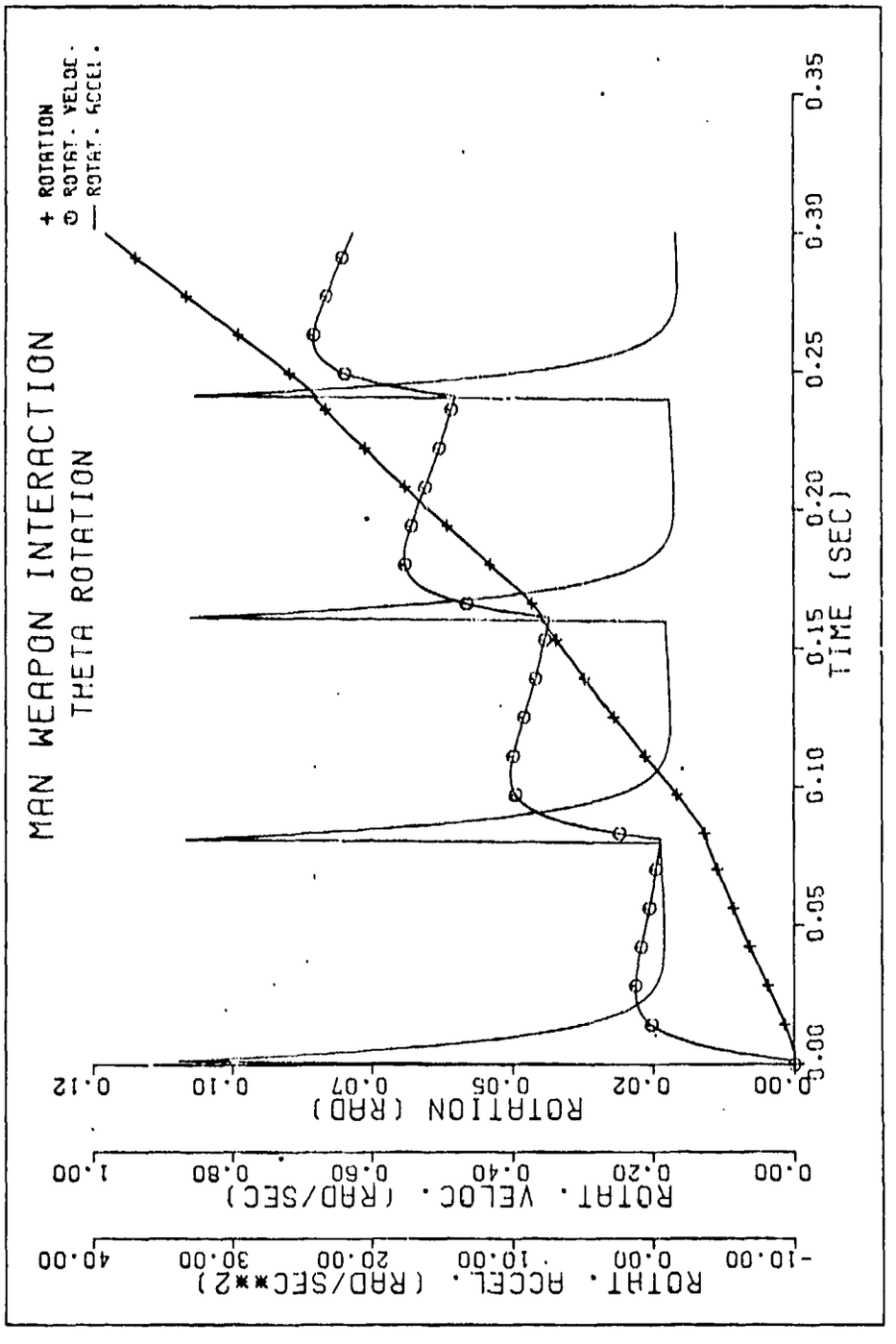


Figure 30 Man Weapon Interaction- Theta Rotation

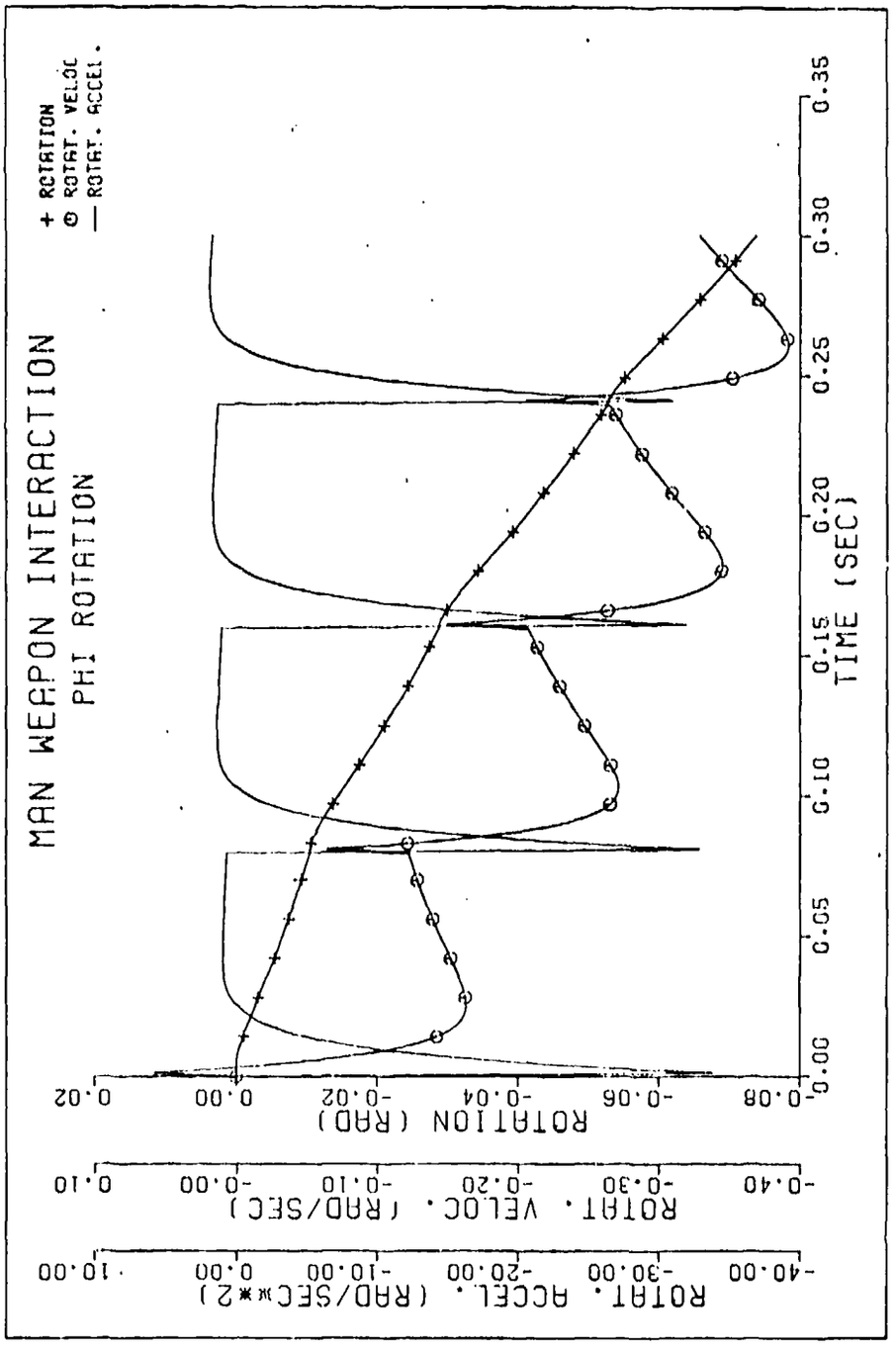


Figure 31 Man Weapon Interaction- Phi Rotation

TABLE 3 SENSITIVITIES OF PARAMETERS THAT AFFECT WEAPON ROTATION

PARAMETER	INCREASED VALUE	RIFLE ROTATION	BEST APPROX.	RIFLE ROTATION	DECREASED VALUE	RIFLE ROTATION
m_r(lb-sec^2/in)	0.05	4.48°	0.099	3.84°	0.2	2.[illegible]
$c_\phi/c_{\phi\ crit}$	0.	3.69°	.5	3.84°	1.	4.02
k_s(lb/in)	5.	5.67°	25.	3.84°	250.	2.99
$c_s/c_{s\ crit}$	0.	5.29°	1.	3.84°	1.5	3.55

4.4 Recommended Design Configurations

Based on experimental findings and model predictions, two possible design configurations for a universal small arms mount are recommended and are illustrated in Figures 32 and 33. By appropriate choices for the masses, lengths, viscous damping constants, and spring stiffnesses, the mount can be designed such that its dynamical characteristics are analogous to the characteristics of the simulation model. The simplified version of the design in Figure 33 is based on the fact that the rotation of the torso mass does not significantly affect the weapon motion or the shoulder reaction force.

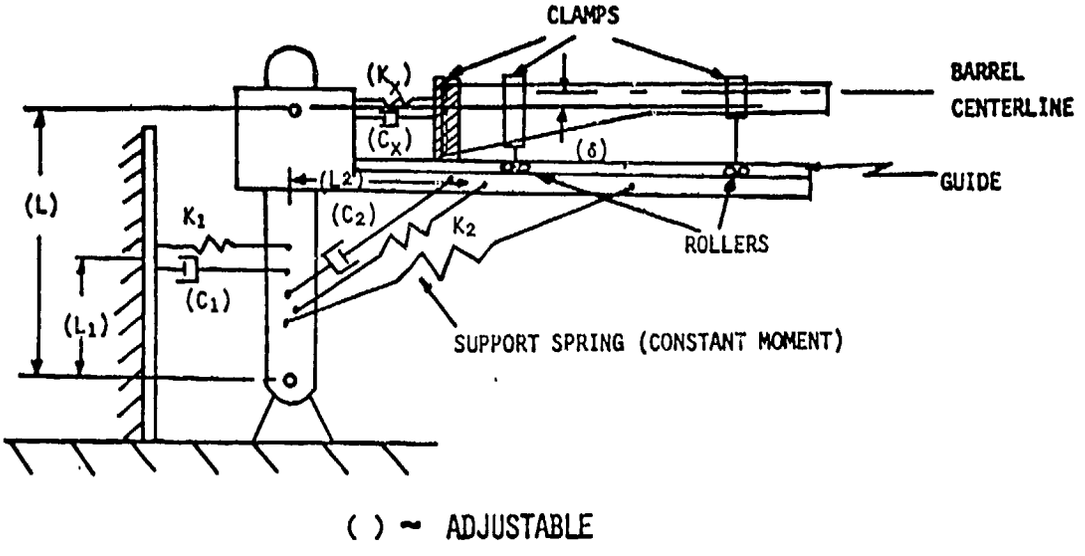


Figure 32 Three Degrees of Freedom Small Arms Mount Concept

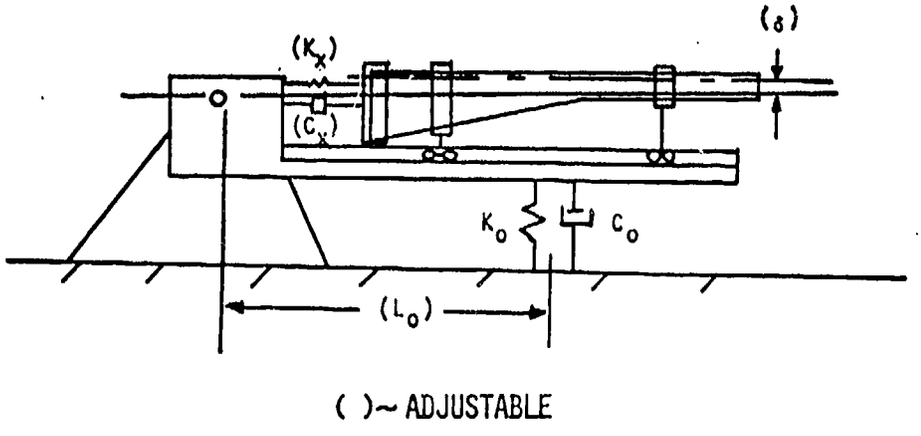


Figure 33 Two Degrees of Freedom Small Arms Mount Concept

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

a. Based on experimental results and model predictions, the force reaction at the shoulder can be approximated by a single linear spring and dashpot. By the proper adjustments of the spring and damper constants, both the impulsive and periodic behavior of the reaction force can be approximated as well as the magnitudes of the peak values.

b. The parameters that affect the peak forces at the shoulder are the shoulder spring stiffness, the shoulder damping coefficient, the impulse of the weapon, and the effective mass of the weapon, arms, and hands. Other model parameters have only a minor influence on the peak values of the shoulder reaction.

c. The weapon rotation after a specified time is not sensitive to the torso inertial or geometric parameters. By effectively removing this link in the model, the weapon rotation is not significantly changed. The parameters that affect the weapon rotation most are the weapon impulse, the eccentricity of the barrel centerline from the shoulder pivot point and the firing frequency. The weapon rotation was found to vary linearly with each of these parameters. Parameters that affect the weapon rotation but are of secondary significance are the effective inertial parameters for the weapon and arms, the torsional stiffness and damping of the weapon pitch, and the shoulder stiffness and damping constants.

d. The recommended configuration for a mount, based on experimental and analytical results, is presented in Figure 33. It allows for two degrees of freedom (pitch and translational) and contains a spring and damper to simulate the shoulder reaction force and an effective torsional spring and damper to simulate the man's torsional stiffness and damping characteristics.

5.2 Recommendations For Future Work

The recommendations for future work in developing a universal small arms mount design are presented below:

a. Additional tests should be performed to support the complete design of a universal small arms mount fixture or the modification of an existing design. The tests would be performed to refine the values obtained for the design parameters.

b. If further tests are conducted, a better method for measuring the shoulder reaction force would be to cut off the end section of weapon stock and insert one or two force washers and an accelerometer directly to the stock. This method would replace the need for the rather heavy and cumbersome shoulder force measuring device shown in Figure 15 and would therefore result in less errors in measurement.

c. Having additional experimental data, refine the mount parameters, using the biodynamics model to best approximate the actual shooter's response. Then incorporate the refined parameter values in the mount design.

d. Investigate existing mount designs to determine if modifications can be made on them to satisfy the design requirements of the recommended concept. If an existing mount is found that can be modified to meet these requirements, then the authors recommend that it be adapted to the design configuration in Figure 33 and tested.

e. Otherwise, if no existing design can be so modified, then a universal small arms mount should be designed from scratch based on the concept shown in Figure 33 and tested.

APPENDIX A

A.0 MODEL REPRESENTATION OF THE HUMAN BODY SEGMENTS

A.1 Head and Torso Representation

The formulas used to calculate the human body segment parameters and the methods for combining the body segments to represent the torso and arms are presented below. The head and trunk model representation is shown in Figure A-1. An ellipsoid of revolution represents the head, while elliptical cylinders represent the trunk. Each of the distances illustrated in Figure A-1 are defined by certain anthropometric dimensions. In particular,

$$a_{hd} = \frac{1}{2} \text{ (stature - cervical height)}$$

$$b_{hd} = \text{(head circumference) } /2\pi,$$

$$a_t = \frac{1}{6} \text{ (chest breadth + waist breadth + hip breadth)}$$

$$b_t = \frac{1}{6} \text{ (chest depth + waist depth + hip depth)}$$

$$\ell_{ut} = \text{cervical height - substernal height,}$$

and

$$\ell_{\ell t} = \text{substernal height - trochanteric height.}$$

The mass and moments of inertia for the head are given by the standard formulas for an ellipsoid of revolution,

$$m_{hd} = \frac{4}{3} \pi a_{hd} b_{hd}^2 \rho_{hd},$$

$$I_{aa_{hd}} = 4 m_{hd} b_{hd}^2,$$

and

$$I_{bb_{hd}} = I_{cc_{hd}} = m_{hd} (a_{hd}^2 + b_{hd}^2) /5$$

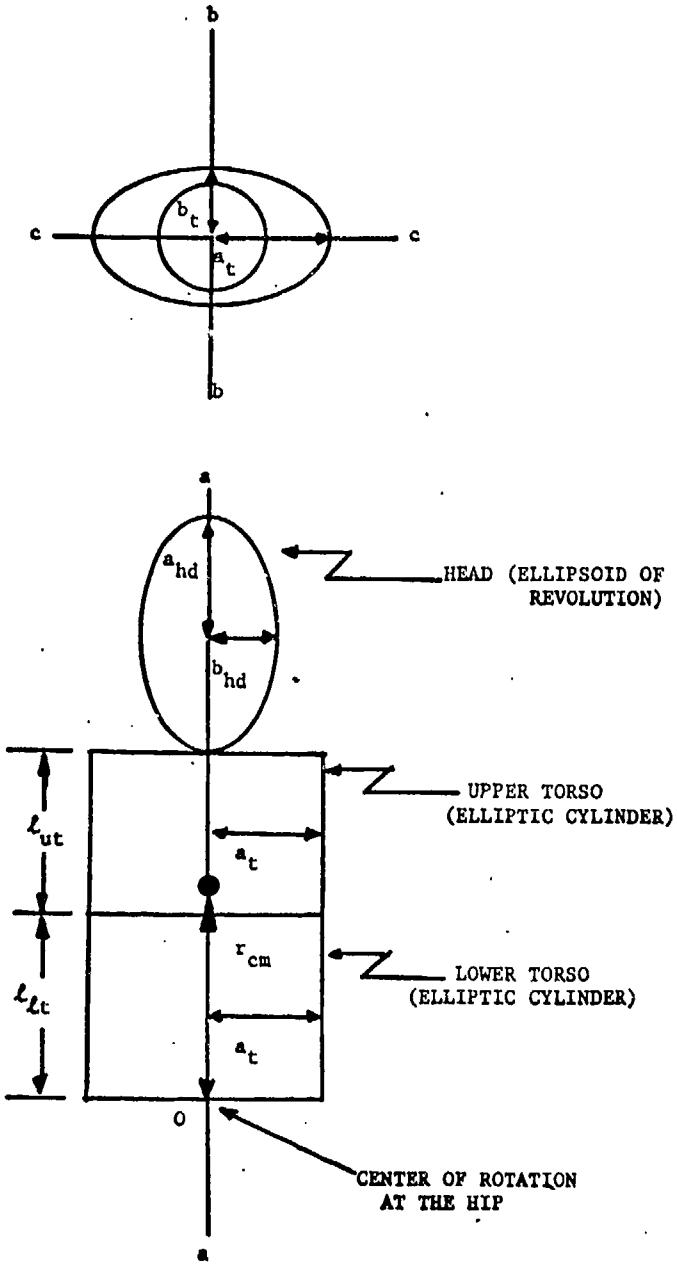


Figure A-1 Head and Torso Rigid Body

and the mass density

$$\rho_{hd} = 1.11 \times 10^3 \qquad (kg/m^3)$$

was obtained from reference[6]. For the upper torso the mass and moments of inertia are given by

$$m_{ut} = \pi\, a_t\, b_t\, \ell_{ut}\, \rho_{ut}$$

$$I_{aa_{ut}} = m_{ut}\, (a_t^2 + b_t^2)\ /4$$

$$I_{bb_{ut}} = m_{ut}\, (3\, a_t^2 + \ell_{ut}^2)\ /12$$

and

$$I_{cc_{ut}} = m_{ut}\, (3\, b_t^2 + \ell_{ut}^2)\ /12$$

Again the mass density

$$\rho_{ut} = 9.2 \times 10^2 \qquad \left(kg/m^3\right)$$

was obtained from reference[6], page 195. Likewise for the lower torso

$$m_{\ell t} = \pi\, a_t\, b_t\, \ell_{\ell t}\, \rho_{\ell t}$$

$$I_{aa_{\ell t}} = m_{\ell t}\, (a_t^2 + b_t^2)\ /4$$

$$I_{bb_{\ell t}} = m_{\ell t}\, (3\, a_t^2 + \ell_{\ell t}^2)\ /12$$

$$I_{cc_{\ell t}} = m_{\ell t}\, (3\, b_t^2 + \ell_{\ell t}^2)\ /12$$

and

$$\rho_{\ell t} = 1.01 \times 10^3 \qquad (kg/m^3)$$

In the model the head, upper torso and lower torso are combined into a single rigid body with mass M,

$$M = m_{hd} + m_{ut} + m_{\ell t}\ .$$

---

[6] Dempster, W.T., "Space Requirements of the Seated Operator," Wright Air Development Center, TR-55-159, Wright-Patterson Air Force Base, Ohio, AD 78792, 1955

Furthermore, the center of mass location, r_{cm}, is given by

$$r_{cm} = [m_{hd} (\ell_{\ell t} + \ell_{ut} + a_{hd}) + m_{ut} (\ell_{\ell t} + 0.5\, \ell_{ut}) + 0.5\, m_{\ell t}\, \ell_{\ell t}] / M$$

and the moment of inertia, I_{cm}, about the center of mass of the head and trunk rigid body is given by,

$$I_{cm} = I_{bb_{hd}} + I_{bb_{ut}} + I_{bb_{\ell t}} + m_{hd} (r_{cm} - \ell_{ut} - \ell_{\ell t} - a_{hd})^2 + m_{ut} (r_{cm} - \ell_{\ell t} - 0.5\, \ell_{ut})^2 + m_{\ell t} (r_{cm} - 0.5\, \ell_{ut})^2$$

A.2 Right Arm, Left Arm and Weapon Configuration

The geometrical representations for the upper arm segments, forearm segments and hands are shown in Figure A-2. The hand is represented by a sphere with density

$$\rho_h = 1.148 \times 10^3 \qquad (kg/m^3)$$

obtained from reference[9] and mass,

$$m_h = 0.01\, W + 0.1588 \qquad (kg)$$

where W = total body weight, obtained from reference[7].
The radius and moment of inertia formulas for the spherical hand are

$$r_h = \frac{(3\, m_h)^{1/3}}{\sqrt[3]{4\, \pi\, \rho_h}}$$

and

$$J_{11_h} = J_{22_h} = J_{33_h} = \frac{2}{5}\, m_h\, r_h^2$$

A frustum of a right circular cone is used to represent both the upper arm segment and the forearm segment. Lenghts ℓ_u and ℓ_ℓ are defined by anthropometric dimensions,

$$\ell_u = \text{acromial height} - \text{radiale height}$$

---

[9]Drillis, R. and Contini, R., "Body Segment Parameters," Office of Vocational Rehabilitation, Department of Health, Education & Welfare, N.Y. University, School of Engineering & Science, N.Y., Report 1166-03, 1966

[7]Barter, J.T., "Estimation of the Mass of Body Segments," Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, TR-57-260, AD 118222, 1957

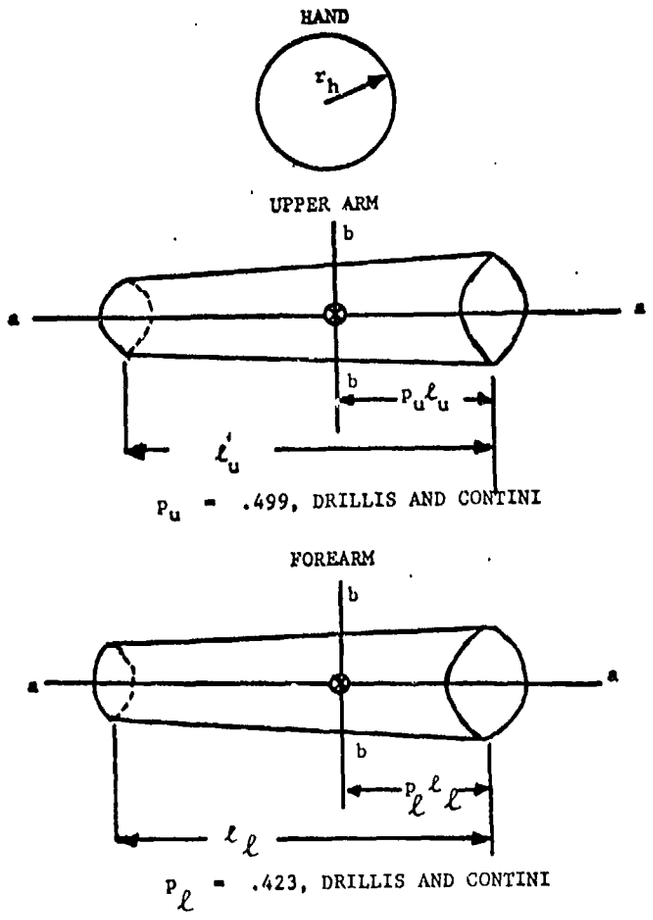


Figure A-2 Geometrical Arm and Hand Representations

and

$$\ell_\ell = \text{radiale height - stylion height}$$

coefficients p_u and p_f were obtained from reference[9];

$$p_u = 0.449$$

$$p_\ell = 0.423$$

and the masses m_u and m_ℓ of the upper arm and forearm, respectively were obtained from reference[7]:

$$m_u = 0.04\ W - 0.6577 \qquad \text{(kg)}$$

and

$$m_\ell = 0.02\ W - 0.1134 \qquad \text{(kg)}$$

where W = total body weight (kg).

The moments of inertia of the upper arm are then given by the standard formulas for a frustrum of a cone,

$$J_{aa_u} = \frac{2\ A_u\ m_u^2}{\rho_a\ \ell_u}$$

and

$$J_{bb_u} = J_{cc_u} = m_u\ [B_u\ \ell_u^2 + A_u\ m_u/(\rho_a\ \ell_u]$$

where

$$A_u = \frac{\kappa_a + \mu_a^3 + \mu_a^4}{20\ \pi\ \kappa_a^2}$$

$$B_u = \frac{3\ [\kappa_a + 3\mu_a\ \kappa_a + (5 + \kappa_a)\ \mu_a^2]}{80\ \kappa_a^2}$$

$$\mu_a = \frac{4\ p_u - 1}{1 - 2\ p_u + \sqrt{12\ p_u\ (1 - p_u) - 2}}$$

and

$$\kappa_a = 1 + \mu_a + \mu_a^2$$

Moments of inertia formulas for the forearm are identifical to those of the upper arm except that the coefficients ℓ_u, p_u and m_u are replaced with ℓ_ℓ, p_ℓ and m_ℓ respectively.

Figure A-3 contains an illustration of the two-dimensional geometrical configuration of the weapon and both arms. Point S locates the shoulder pivot point. For convenience the assumption is made that both arms are connected at point S. The right hand grasps the weapon at distance c_1 and the left hand grasps the weapon at distance c_2 and distance x_w locates the center mass of the weapon. Table A-1 contains the weapon mass, moment of inertia and center of gravity location of the M16 rifle and the M203 grenade launcher, as measured by Mr. T. Hutchings.

TABLE A-1 WEAPON PARAMETERS

PARAMETER	M-16	M203
m_w (lb)	8.6	9.4
I_w(lb-in-sec^2)	1.6	3.6
X_w (in)	17.	19.

Referring to Figure A-3, the total mass of the arm-weapon system and the center of mass location (x_o, y_o) are given by

$$m_r = 2\, m_u + 2\, m_\ell + 2\, m_h + m_w$$

$$x_o = [m_u\, p_u\, \ell_u\, (\cos\alpha + \cos\beta) + m_\ell\, (\ell_u + p_\ell\, \ell_\ell)\, (\cos\alpha + \cos\beta) + m_h\, (c_1 + c_2) + m_w\, x] / m_r$$

and

$$y_o = [m_u\, p_u\, \ell_u + m_\ell\, (\ell_u + p_\ell\, \ell_\ell)]\, (\sin\alpha + \sin\beta)\, /m_r$$

where

$$\cos\beta = \frac{\ell_u^2 + c_1^2 - \ell_\ell^2}{2\, \ell_u\, c_1}$$

$$\cos\alpha = \frac{\ell_u^2 + c_2^2 - \ell_\ell^2}{2\, \ell_u\, c_2}$$

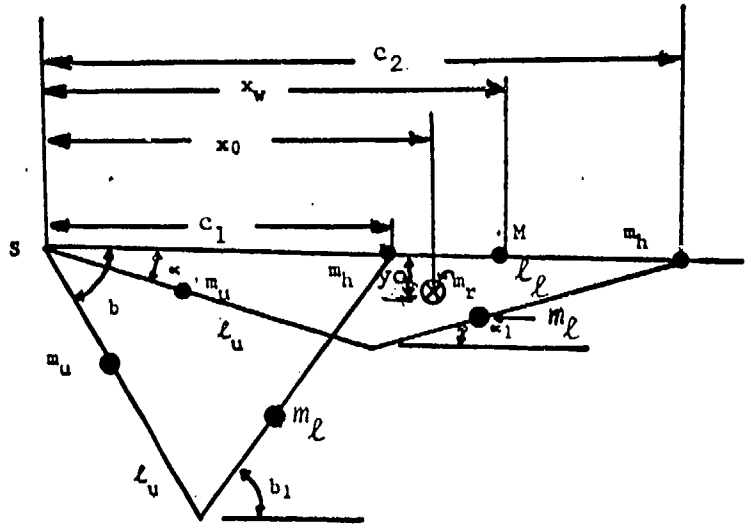


Figure A-3 Weapon and Arm Rigid Body

Finally the combined moment of inertia of the arm-weapon system about the center of mass becomes

$$
\begin{aligned}
J_{cm} = {} & 2\,J_{11_h} + 2\,J_{bb_u} + 2\,J_{bb_\ell} + J_w \\
& + m_h\,[(x_o - c_1)^2 + (x_o - c_2)^2 + y_o^2] \\
& + m_u\,[(x_o - p_u\,\ell_u \cos\alpha)^2 + (x_o - p_u\,\ell_u \cos\beta)^2 \\
& + (y_o - p_u\,\ell_u \sin\alpha)^2 + (y_o - p_u\,\ell_u \sin\beta)^2\,] \\
& + m_\ell\,\{[x_o - (\ell_u + p_\ell\,\ell_\ell)\cos\alpha]^2 + [x_o - (\ell_u + p_\ell\,\ell_\ell)\cos\beta]^2 \\
& + [y_o - (\ell_u - p_\ell\,\ell_\ell)\sin\alpha]^2 + [y_o - (\ell_u - p_\ell\,\ell_\ell)\sin\beta]^2\} \\
& + m_w\,[(x_o - x_w)^2 + y_o]
\end{aligned}
$$

A.3 Anthropometric Data and Calculated Results

A list is provided, in Table A-2, of some typical weights and anthropometric dimensions for three men of various sizes (small, medium, large). This table contains a partial list of data that were measured by Santschi reference[8]. Table A-3 contains the calculated dimensions and inertial parameters based on the data in Table A-1 and A-2. The arm-weapon inertial parameter, I_{cm}, is somewhat larger than actual because the five pound shoulder force measuring device was included in the inertia and center of mass calculations. The results in Table A-3 show the orders of magnitude of the model parameters along with the expected variations for different size men.

---

[8]Santschi, W.R., DuBois, J., and Omoto, C., "Moments of Inertia and Centers of Gravity of the Living Human Body, Aerospace Medical Research Laboratories, Wright-Patterson Air Force Base, Ohio, AMRL-TDR-63-66, AD 410451, 1963

TABLE A-2 (SANTSCHI)
WEIGHT AND ANTHROPOMETRIC DATA FOR THREE TEST SUBJECTS

	SUBJECT 1	SUBJECT 2	SUBJECT 3
Weight	161.6 (lb)	172.2 (lb)	210.4 (lb)
Stature	66.9 (in)	73.0 (in)	72.3 (in)
Head Circ.	22.2	23.5	23.3
Chest Breadth	13.3	12.7	15.6
Waist Breadth	11.6	12.0	13.2
Hip Breadth	13.8	14.8	16.3
Chest Depth	9.3	10.3	10.4
Waist Depth	8.3	9.3	10.9
Hip Depth	10.5	10.6	10.5
Cervical ht.	56.3	62.7	62.4
Substernal ht.	47.8	50.8	50.7
Trochanteric ht.	33.5	39.2	38.5
Forearm Length	9.8	11.5	11.1
Upper Arm Length	12.8	13.7	14.0

TABLE A-3 CALCULATED BIOMECHANICAL PARAMETERS
FOR THREE TEST SUBJECTS

	SUBJECT #1	SUBJECT #2	SUBJECT #3
M (lb)	87.	97.	116.
I_{cm} (lb-in-sec^2)	18.	21.	26.
gm_r (lb)	32.	33.	38.
I_r (lb-in-sec^2)	9.7	11.	13.
L (in)	20.	21.	21.
r_{cm} (in)	13.	14.	13.
x_o (in)	9.6	9.5	9.3

APPENDIX B

B.0 DIGITAL COMPUTER PROGRAM LISTING

```
      REAL M,MR,ICM,IR,L,KXW,KXS,KT,KP,MU,NU,K,IF
      COMMON /$A/ M,MR,ICM,IR,L,D,XS,RCM,KXW,KXS,KT,KP,CDX,
     * CDT,CDP,G,K,XO,TO,PO,STO,STPO,CTPO,TIM
      COMMON /$PLOT/ NP,TIME(502),R(502),Z(502)
      DIMENSION PRMT(5),RHS(6),Y(6),AUX(8,6),ERW(6)
      EXTERNAL FCT,OUTP
      CALL PLOTS(IBUF,IDUM,14)
C
C        ALL UNITS ARE IN THE POUND, INCH, SECOND SYSTEM
C
      G = 386.4
      NDIM = 6
      NRUN = 0
C
C        READ INERTIA DATA
C
   10 READ (5,1000,END=999) M,MR,ICM,IR
      NP = 0
      NRUN = NRUN + 1
C
C        READ DISTANCES
C
      READ (5,1000) L,D,XS,RCM
C
C        READ SPRING CONSTANTS
C
      READ (5,1000) KXW,KXS,KT,KP
C
C        READ DAMPING RATIOS
C
      READ (5,1000) CRX,CRT,CRP
C
C        CALCULATE DAMPING COEFFICIENTS
C
      CDX = CRX*2.*SQRT(KXS*MR)
      CDT = CRT*2.*SQRT(KT*ICM)
      CDP = CRP*2.*SQRT(KP*IR)
C
C        READ INTEGRATION PARAMETERS:  START TIME, STOP TIME,
C        TIME INTERVAL, UPPER ERROR BOUND, AND PRINT PARAMETER
C
      READ (5,1000) (PRMT(I),I=1,4)
C
C        READ INITIAL CONDITIONS:  XDOT(1-3), X(1-3)
C
```

```
      READ (5,1000) Y
C
C        READ ERROR WEIGHTS FOR RKGS
C
      READ (5,1000) ERW
C
C        READ SHOT DURATION, AND THE TIME BETWEEN SHOTS AND WEAPON IMPULSE
C
      READ (5,1000) TDUR,TINT,IF
C
C        PRINT INPUT DATA
C
      WRITE (6,2000) NRUN
      WRITE(6,3000) M,MR,ICM,IR
      WRITE(6,4000)
      WRITE(6,3000) L,D,XS,RCM
      WRITE(6,5000)
      WRITE(6,3000) KXW,KXS,KT,KP
      WRITE(6,6000)
      WRITE(6,3000) CRX,CRT,CRP
      WRITE(6,7000)
      WRITE(6,3000) (PRMT(I),I=1,4)
      WRITE(6,8000)
      WRITE(6,3000) (Y(I),I=1,3)
      WRITE(6,9000)
      WRITE(6,3000) (Y(I),I=4,6)
      WRITE(6,5500)
      WRITE(6,3000) (ERW(I),I=1,3)
      WRITE(6,6500)
      WRITE(6,3000) (ERW(I),I=4,6)
      WRITE(6,7500)
      WRITE(6,3000) TDUR,TINT,IF
C
C        WRITE HEADING FOR OUTP
C
      WRITE(6,1600)
C
C        INITIALIZE PARAMETERS
C
      TEND = PRMT(2)
      K = ICM + M*RCM**2
C
C        CALCULATE INITIAL CONDITIONS AFTER THE NEXT SHOT
C
      T = PRMT(1)
```

```
      XO = Y(4)
      TO = Y(5)
      PO = Y(6)
      STO = SIN(TO)
      CTO = COS(TO)
      SPO = SIN(PO)
      CPO = COS(PO)
      STPO = SIN(TO + PO)
      CTPO = COS(TO + PO)
      MU = MR*L*CPO
      DL = L*CPO + D
      CALL OUTP(T,Y,RHS,IHLF,NDIM,PRMT)
   20 PRMT(1) = TIM + TDUR
      PRMT(2) = TIM + TINT
      IF(PRMT(1).GE.TEND) GO TO 50
      IF(PRMT(2).GT.TEND) PRMT(2) = TEND
      DO 25 I=1,3
   25 RHS(I) = ERW(I)
      ETA = Y(4) + L*SPO
      NU = IR + MR*Y(4)**2
      B11 = K*NU + MR*IR*L**2 + (MU*Y(4))**2
      B12 = NU*MU
      B13 = -(IR + MR*Y(4)*ETA)*MU
      B22 = MR*NU
      B23 = -MR*(IR + MR*Y(4)*ETA)
      B33 = MR*(K + IR + MR*ETA**2)
      DELTA = K*MR*NU + IR*(MR*L*SPO)**2
      Y(1) = Y(1) + IF*(-B11 + B12*DL + B13*D)/DELTA
      Y(2) = Y(2) + IF*(-B12 + B22*DL + B23*D)/DELTA
      Y(3) = Y(3) + IF*(-B13 + B23*DL + B33*D)/DELTA
C
C        INTEGRATE THE MAN/WEAPON EQUATIONS
C
      CALL RKGS(PRMT,Y,RHS,NDIM,IHLF,FCT,OUTP,AUX)
      WRITE(6,2500) IHLF
      GO TO 20
   50 WRITE(6,3500)
      DO 60 I=1,NP
      WRITE(6,4500) I,TIME(I),R(I),Z(I)
   60 CONTINUE
C
C        PLOT RESULTS
C
      CALL CALPLT(NRUN)
      GO TO 10
```

```
 999 CALL PLOT(12.0,0.0,999)
1000 FORMAT(8F10.0)
1500 FORMAT('1',T4,'TIME',T17,'XDOT',T37,'TDOT',T57,'PDOT',T77,'X',
    * T97,'T',T117,'P')
2000 FORMAT('1THREE DEGREE OF FREEDOM MAN/WEAPON MODEL INPUT',5X,'RUN '
    * ,I3/
    * 'OINERTIA DATA:'/T3,'M',T28,'MR',T53,'ICM',T78,'IR')
2500 FORMAT('OIHLF =',I3//)
3000 FORMAT(1X,4(E15.8,10X))
3500 FORMAT('1 NP',T10,'TIME',T30,'RECOIL',T50,'PITCH')
4000 FORMAT('ODISTANCES:'/T3,'L',T28,'D',T53,'XS',T78,'RCM')
4500 FORMAT(' ',I3,T10,F8.5,T30,E15.8,T50,E15.8)
5000 FORMAT('OSPRING CONSTANTS:'/T3,'KXW',T28,'KXS',T53,'KT',T78,'KP')
5500 FORMAT('OVELOCITY ERROR WEIGHTS:'/T3,'XDOT',T28,'TDOT',T53,'PDOT')
6000 FORMAT('OCRITICAL DAMPING RATIOS:'/T3,'CRX',T28,'CRT',T53,'CRP')
6500 FORMAT('OPOSITION ERROR WEIGHTS:'/T3,'X',T28,'T',T53,'P')
7000 FORMAT('ORKGS PARAMETERS:'/T3,'START TIME',T28,'STOP TIME',
    * T53,'TIME INCR',T78,'ERROR BOUND')
7500 FORMAT('OTIME PARAMETERS:'/T3,'TOUR',T28,'TINT',T53,'IF')
8000 FORMAT('OINITIAL VELOCITIES:'/T3,'XDOT',T28,'TDOT',T53,'PDOT')
9000 FORMAT('OINITIAL POSITIONS:'/T3,'X',T28,'T',T53,'P')
     STOP
     END
```

```
      SUBROUTINE CALPLT(NRUN)
      COMMON /$PLOT/ NP,TIME(502),R(502),Z(502)
      DIMENSION T(4),DAT(7)
      T(1) = TIME(1)
      T(2) = TIME(NP)
      FRN = NRUN
C
C        PREPARE FOR THE FIRST PLOT
C
      CALL NEWPEN(1)
      CALL PLOT(0.0,-15.0,-3)
      CALL DASHP(0.0,16.0,0.3)
      CALL PLOT(1.0,-15.0,-3)
      CALL PLOT(0.0,3.0,-3)
      CALL RECT(0.0,0.0,7.5,9.0,0.0,2)
      CALL PLOT(1.5,0.75,-3)
      CALL SCALE(T,7.0,2,1)
      CALL SCALE(R,5.25,NP,1)
      CALL SCALE(Z,5.25,NP,1)
      TIME(NP+1) = T(3)
      TIME(NP+2) = T(4)
C
C        DRAW AXES AND GRAPHS
C
      CALL AXIS(0.0,0.0,'TIME (SECONDS)',-14,7.0,0.0,T(3),T(4))
      CALL AXIS(0.0,0.0,'PITCH ANGLE (DEGREES)',21,5.25,90.0,Z(NP+1),
     * Z(NP+2))
      ZEROP = ABS(Z(NP+1)/Z(NP+2))
      CALL PLOT(0.0,ZEROP,3)
      CALL PLOT(7.0,ZEROP,2)
      CALL LINE(TIME,Z,NP,1,0,1)
      CALL NEWPEN(2)
      CALL AXIS(-0.75,0.0,'MOUNT FORCE (POUNDS)',20,5.25,90.0,
     * R(NP+1),R(NP+2))
      ZEROP = ABS(R(NP+1)/R(NP+2))
      CALL PLOT(-0.75,ZEROP,3)
      CALL PLOT(7.00,ZEROP,2)
      CALL LINE(TIME,R,NP,1,0,1)
C
C        WRITE TITLE HEADINGS
C
      CALL NEWPEN(1)
      CALL SYMBOL(1.5,6.25,0.21,'MAN/WEAPON ANALYSIS',0.0,19)
      CALL SYMBOL(1.5,6.0,0.14,'MOUNT FORCE AND TOTAL PITCH ANGLE',
     * 0.0,33)
```

```
      CALL GBDATE(DAT)
      CALL SYMBOL(1.5,5.75,0.14,DAT,0.0,28)
      CALL SYMBOL(1.5,5.50,0.14,'RUN ',0.0,4)
      CALL NUMBER(999.,999.,0.14,FRN,0.0,0)
C
C        PREPARE FOR THE NEXT PLOT
C
      CALL PLOT(8.5,-15.0,-3)
      CALL DASHP(0.0,16.0,0.3)
      RETURN
      END
```

```
      SUBROUTINE FCT(T,Y,RHS)
      REAL M,MR,ICM,IR,L,KXW,KXS,KT,KP,MU,NU,K
      COMMON /$A/ M,MR,ICM,IR,L,D,XS,RCM,KXW,KXS,KT,KP,CDX,
     * CDT,CDP,G,K,XO,TO,PO,STO,STPO,CTPO,TIM
      DIMENSION Y(6),X(3),XD(3),RHS(6)
      DO 10 I=1,3
      J=I+3
      X(I) = Y(J)
   10 XD(I) = Y(I)
      STP = SIN(X(2) + X(3))
      CTP = COS(X(2) + X(3))
      ST = SIN(X(2))
      CT = COS(X(2))
      SP = SIN(X(3))
      CP = COS(X(3))
      MU = MR*L*CP
      ETA = X(1) + L*SP
      NU = IR + MR*X(1)**2
      B11 = K*NU + MR*IR*L**2 + (MU*X(1))**2
      B12 = NU*MU
      B13 = -(IR + MR*X(1)*ETA)*MU
      B22 = MR*NU
      B23 = -MR*(IR + MR*X(1)*ETA)
      B33 = MR*(K + IR + MR*ETA**2)
      DELTA = K*MR*NU + IR*(MR*L*SP)**2
      FX = MR*X(1)*(XD(2) + XD(3))**2 + MR*L*SP*XD(2)**2 +MR*G*(STPO
     * -STP)
      FKX = KXW*(X(1) - XO) + CDX *XD(1)
      IF(X(1).LT.XS) FKX = KXS*(X(1)-XS)+KXW*(XS-XO)+CDX *XD(1)
      FX = FX - FKX
      FT = -2.*MR*ETA*XD(1)*(XD(2)+XD(3))-X(1)*MU*XD(3)*(2.*XD(2)+XD(3))
     * -CDT*XD(2)-KT*(X(2)-TO)                +(M*RCM+MR*L)*G*(ST-STO)
     * -MR*G*(X(1)*CTP-XO*CTPO)
      FP = -2.*MR*X(1)*XD(1)*(XD(2)+XD(3))+MU*XD(1)*XD(2)**2-CDP*XD(3)
     * -KP*(X(3)-PO)-MR*G*X(1)*CTP        +MR*G*XO*CTPO
      RHS(1) = (FX*B11 + FT*B12 + FP*B13)/DELTA
      RHS(2) = (FX*B12 + FT*B22 + FP*B23)/DELTA
      RHS(3) = (FX*B13 + FT*B23 + FP*B33)/DELTA
      RHS(4) = XD(1)
      RHS(5) = XD(2)
      RHS(6) = XD(3)
      RETURN
      END
```

```
      SUBROUTINE OUTP(T,Y,RHS,IHLF,NDIM,PRMT)
      REAL M,MR,ICM,IR,L,KXW,KXS,KT,KP,MU,NU,K
      COMMON /$A/ M,MR,ICM,IR,L,D,XS,RCM,KXW,KXS,KT,KP,COX,
     * CDT,CDP,G,K,XO,TO,PO,STO,STPO,CTPO,TIM
      COMMON /$PLOT/ NP,TIME(502),R(502),Z(502)
      DIMENSION Y(6),RHS(6),PRMT(5)
      TIM = T
      CPI = 180./3.1415927
      NP = NP + 1
      IF(NP.GT.500) GO TO 50
      R(NP) = KXW*(Y(4) - XO) + COX *Y(1)
      IF(Y(4).LT.XS) R(NP) = KXS*(Y(4) - XS) + KXW*(XS - XO) + COX *Y(1)
      Z(NP) = (Y(5) + Y(6))*CPI
      TIME(NP) = T
   50 CONTINUE
C **********  ADDITION FOR OTHER PLOTS  **********
      WRITE(3)T,Y,RHS
      WRITE(6,1000) T,Y
 1000 FORMAT(1X,F8.5,T10,6(5X,E15.8))
      RETURN
      END
```

```
C                                                                       RKGS  10
C     ...................................................................RKGS  20
C                                                                       RKGS  30
C        SUBROUTINE RKGS                                                RKGS  40
C                                                                       RKGS  50
C        PURPOSE                                                        RKGS  60
C           TO SOLVE A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL      RKGS  70
C           EQUATIONS WITH GIVEN INITIAL VALUES.                        RKGS  80
C                                                                       RKGS  90
C        USAGE                                                          RKGS 100
C           CALL RKGS (PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)              RKGS 110
C           PARAMETERS FCT AND OUTP REQUIRE AN EXTERNAL STATEMENT.      RKGS 120
C                                                                       RKGS 130
C        DESCRIPTION OF PARAMETERS                                      RKGS 140
C           PRMT   - AN INPUT AND OUTPUT VECTOR WITH DIMENSION GREATER  RKGS 150
C                    OR EQUAL TO 5, WHICH SPECIFIES THE PARAMETERS OF   RKGS 160
C                    THE INTERVAL AND OF ACCURACY AND WHICH SERVES FOR  RKGS 170
C                    COMMUNICATION BETWEEN OUTPUT SUBROUTINE (FURNISHED RKGS 180
C                    BY THE USER) AND SUBROUTINE RKGS. EXCEPT PRMT(5)   RKGS 190
C                    THE COMPONENTS ARE NOT DESTROYED BY SUBROUTINE     RKGS 200
C                    RKGS AND THEY ARE                                  RKGS 210
C           PRMT(1)- LOWER BOUND OF THE INTERVAL (INPUT).               RKGS 220
C           PRMT(2)- UPPER BOUND OF THE INTERVAL (INPUT).               RKGS 230
C           PRMT(3)- INITIAL INCREMENT OF THE INDEPENDENT VARIABLE      RKGS 240
C                    (INPUT).                                           RKGS 250
C           PRMT(4)- UPPER ERROR BOUND (INPUT). IF ABSOLUTE ERROR IS    RKGS 260
C                    GREATER THAN PRMT(4), INCREMENT GETS HALVED.       RKGS 270
C                    IF INCREMENT IS LESS THAN PRMT(3) AND ABSOLUTE     RKGS 280
C                    ERROR LESS THAN PRMT(4)/50, INCREMENT GETS DOUBLED.RKGS 290
C                    THE USER MAY CHANGE PRMT(4) BY MEANS OF HIS        RKGS 300
C                    OUTPUT SUBROUTINE.                                 RKGS 310
C           PRMT(5)- NO INPUT PARAMETER. SUBROUTINE RKGS INITIALIZES    RKGS 320
C                    PRMT(5)=0. IF THE USER WANTS TO TERMINATE          RKGS 330
C                    SUBROUTINE RKGS AT ANY OUTPUT POINT, HE HAS TO     RKGS 340
C                    CHANGE PRMT(5) TO NON-ZERO BY MEANS OF SUBROUTINE  RKGS 350
C                    OUTP. FURTHER COMPONENTS OF VECTOR PRMT ARE        RKGS 360
C                    FEASIBLE IF ITS DIMENSION IS DEFINED GREATER       RKGS 370
C                    THAN 5. HOWEVER SUBROUTINE RKGS DOES NOT REQUIRE   RKGS 380
C                    AND CHANGE THEM. NEVERTHELESS THEY MAY BE USEFUL   RKGS 390
C                    FOR HANDING RESULT VALUES TO THE MAIN PROGRAM       RKGS 400
C                    (CALLING RKGS) WHICH ARE OBTAINED BY SPECIAL       RKGS 410
C                    MANIPULATIONS WITH OUTPUT DATA IN SUBROUTINE OUTP. RKGS 420
C           Y      - INPUT VECTOR OF INITIAL VALUES. (DESTROYED)        RKGS 430
C                    LATERON Y IS THE RESULTING VECTOR OF DEPENDENT     RKGS 440
C                    VARIABLES COMPUTED AT INTERMEDIATE POINTS X.       RKGS 450
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C            DERY   - INPUT VECTOR OF ERROR WEIGHTS.  (DESTROYED)        RKGS 460
C                     THE SUM OF ITS COMPONENTS MUST BE EQUAL TO 1.       RKGS 470
C                     LATERON DERY IS THE VECTOR OF DERIVATIVES. WHICH    RKGS 480
C                     BELONG TO FUNCTION VALUES Y AT A POINT X.           RKGS 490
C            NDIM   - AN INPUT VALUE, WHICH SPECIFIES THE NUMBER OF       RKGS 500
C                     EQUATIONS IN THE SYSTEM.                            RKGS 510
C            IHLF   - AN OUTPUT VALUE, WHICH SPECIFIES THE NUMBER OF      RKGS 520
C                     BISECTIONS OF THE INITIAL INCREMENT. IF IHLF GETS    RKGS 530
C                     GREATER THAN 10, SUBROUTINE RKGS RETURNS WITH       RKGS 540
C                     ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM. ERROR      RKGS 550
C                     MESSAGE IHLF=12 OR IHLF=13 APPEARS IN CASE          RKGS 560
C                     PRMT(3)=0 OR IN CASE SIGN(PRMT(3)).NE.SIGN(PRMT(2)-RKGS 570
C                     PRMT(1)) RESPECTIVELY.                              RKGS 580
C            FCT    - THE NAME OF AN EXTERNAL SUBROUTINE USED. THIS      RKGS 590
C                     SUBROUTINE COMPUTES THE RIGHT HAND SIDES DERY OF    RKGS 600
C                     THE SYSTEM TO GIVEN VALUES X AND Y. ITS PARAMETER   RKGS 610
C                     LIST MUST BE X,Y,DERY. SUBROUTINE FCT SHOULD        RKGS 620
C                     NOT DESTROY X AND Y.                                RKGS 630
C            OUTP   - THE NAME OF AN EXTERNAL OUTPUT SUBROUTINE USED.     RKGS 640
C                     ITS PARAMETER LIST MUST BE X,Y,DERY,IHLF,NDIM,PRMT. RKGS 650
C                     NONE OF THESE PARAMETERS (EXCEPT, IF NECESSARY,     RKGS 660
C                     PRMT(4),PRMT(5),...) SHOULD BE CHANGED BY           RKGS 670
C                     SUBROUTINE OUTP. IF PRMT(5) IS CHANGED TO NON-ZERO. RKGS 680
C                     SUBROUTINE RKGS IS TERMINATED.                      RKGS 690
C            AUX    - AN AUXILIARY STORAGE ARRAY WITH 8 ROWS AND NDIM    RKGS 700
C                     COLUMNS.                                            RKGS  10
C                                                                         RKGS 720
C        REMARKS                                                          RKGS 730
C           THE PROCEDURE TERMINATES AND RETURNS TO CALLING PROGRAM, IF   RKGS 740
C           (1) MORE THAN 10 BISECTIONS OF THE INITIAL INCREMENT ARE      RKGS 750
C               NECESSARY TO GET SATISFACTORY ACCURACY (ERROR MESSAGE     RKGS 760
C               IHLF=11).                                                 RKGS 770
C           (2) INITIAL INCREMENT IS EQUAL T'    OR HAS WRONG SIGN        RKGS 780
C               (ERROR MESSAGES IHLF=12 OR IHLF=13).                      RKGS  9Z
C           (3) THE WHOLE INTEGRATION INTERVAL IS WORKED THROUGH.         RKGS 800
C           (4) SUBROUTINE OUTP HAS CHANGED PRMT(5) TO NON-ZERO.          RKGS 810
C                                                                         RKGS 820
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                    RKGS 830
C           THE EXTERNAL SUBROUTINES FCT(X,Y,DERY) AND                    RKGS 840
C           OUTP(X,Y,DERY,IHLF,NDIM,PRMT) MUST BE FURNISHED BY THE USER.  RKGS 850
C                                                                         RKGS 860
C        METHOD                                                           RKGS 870
C           EVALUATION IS DONE BY MEANS OF FOURTH ORDER RUNGE-KUTTA       RKGS 880
C           FORMULAE IN THE MODIFICATION DUE TO GILL. ACCURACY IS         RKGS 890
C           TESTED COMPARING THE RESULTS OF THE PROCEDURE WITH SINGLE     RKGS 900
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C           AND DOUBLE INCREMENT.                                          RKGS 910
C           SUBROUTINE RKGS AUTOMATICALLY ADJUSTS THE INCREMENT DURING     RKGS 920
C           THE WHOLE COMPUTATION BY HALVING OR DOUBLING. IF MORE THAN     RKGS 930
C           10 BISECTIONS OF THE INCREMENT ARE NECESSARY TO GET            RKGS 940
C           SATISFACTORY ACCURACY, THE SUBROUTINE RETURNS WITH             RKGS 950
C           ERROR MESSAGE IHLF=11 INTO MAIN PROGRAM.                       RKGS 960
C           TO GET FULL FLEXIBILITY IN OUTPUT, AN OUTPUT SUBROUTINE        RKGS 970
C           MUST BE FURNISHED BY THE USER.                                 RKGS 980
C           FOR REFERENCE, SEE                                             RKGS 990
C           RALSTON/WILF, MATHEMATICAL METHODS FOR DIGITAL COMPUTERS,      RKGS1000
C           WILEY, NEW YORK/LONDON, 1960, PP.110-120.                      RKGS1010
C                                                                          RKGS1020
C     ..................................................................RKGS1030
C                                                                          RKGS1040
      SUBROUTINE RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)                  RKGS1050
C                                                                          RKGS1060
C                                                                          RKGS1070
      DIMENSION Y(1),DERY(1),AUX(8,1),A(4),B(4),C(4),PRMT(1)
      DO 1 I=1,NDIM                                                        RKGS1090
    1 AUX(8,I)=.06666667*DERY(I)                                           RKGS1100
      X=PRMT(1)                                                            RKGS1110
      XEND=PRMT(2)                                                         RKGS1120
      H=PRMT(3)                                                            RKGS1130
      PRMT(5)=0.                                                           RKGS1140
      CALL FCT(X,Y,DERY)                                                   RKGS1150
C                                                                          RKGS1160
C     ERROR TEST                                                           RKGS1170
      IF(H*(XEND-X))38,37,2                                                RKGS1180
C                                                                          RKGS1190
C     PREPARATIONS FOR RUNGE-KUTTA METHOD                                  RKGS1200
    2 A(1)=.5                                                              RKGS1210
      A(2)=.2928932                                                        RKGS1220
      A(3)=1.707107                                                        RKGS1230
      A(4)=.1666667                                                        RKGS1240
      B(1)=2.                                                              RKGS1250
      B(2)=1.                                                              RKGS1260
      B(3)=1.                                                              RKGS1270
      B(4)=2.                                                              RKGS1280
      C(1)=.5                                                              RKGS1290
      C(2)=.2928932                                                        RKGS1300
      C(3)=1.707107                                                        RKGS1310
      C(4)=.5                                                              RKGS1320
C                                                                          RKGS1330
C     PREPARATIONS OF FIRST RUNGE-KUTTA STEP                               RKGS1340
      DO 3 I=1,NDIM                                                        RKGS1350
```

```
      AUX(1,I)=Y(I)                                                    RKGS1360
      AUX(2,I)=DERY(I)                                                 RKGS13 0
      AUX(3,I)=0.                                                      RKGS1380
    3 AUX(6,I)=0.                                                      RKGS1390
      IREC=0                                                           RKGS1400
      H=H+H                                                            RKGS1410
      IHLF=-1                                                          RKGS1420
      ISTEP=0                                                          RKGS1430
      IEND=0                                                           RKGS1440
C                                                                      RKGS1450
C                                                                      RKGS1460
C     START OF A RUNGE-KUTTA STEP                                      RKGS1470
    4 IF((X+H-XEND)*H)7,6,5                                            RKGS1480
    5 H=XEND-X                                                         RKGS1490
    6 IEND=1                                                           RKGS1500
C                                                                      RKGS1510
C     RECORDING OF INITIAL VALUES OF THIS STEP                         RKGS1520
    7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)                               RKGS1530
      IF(PRMT(5))40,8,40                                               RKGS1540
    8 ITEST=0                                                          RKGS1550
    9 ISTEP=ISTEP+1                                                    RKGS1560
C                                                                      RKGS1570
C                                                                      RKGS1580
C     START OF INNERMOST RUNGE-KUTTA LOOP                              RKGS1590
      J=1                                                              RKGS1600
   10 AJ=A(J)                                                          RKGS1610
      BJ=B(J)                                                          RKGS1620
      CJ=C(J)                                                          RKGS1630
      DO 11 I=1,NDIM                                                   RKGS1640
      R1=H*DERY(I)                                                     RKGS1650
      R2=AJ*(R1-BJ*AUX(6,I))                                           RKGS1660
      Y(I)=Y(I)+R2                                                     RKGS1670
      R2=R2+R2+R2                                                      RKGS1680
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1                                       RKGS1690
      IF(J-4)12,15,15                                                  RKGS1700
   12 J=J+1                                                            RKGS1 10
      IF(J-3)13,14,13                                                  RKGS1720
   13 X=X+.5*H                                                         RKGS1730
   14 CALL FCT(X,Y,DERY)                                               RKGS1740
      GOTO 10                                                          RKGS1750
C     END OF INNERMOST RUNGE-KUTTA LOOP                                RKGS1760
C                                                                      RKGS1770
C                                                                      RKGS1780
C     TEST OF ACCURACY                                                 RKGS1790
   15 IF(ITEST)16,16,20                                                RKGS1800
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```
C                                                                       RKGS1810
C     IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY   RKGS1820
   16 DO 17 I=1,NDIM                                                    RKGS1830
   17 AUX(4,I)=Y(I)                                                     RKGS1840
      ITEST=1                                                           RKGS1850
      ISTEP=ISTEP+ISTEP-2                                               RKGS1860
   18 IHLF=IHLF+1                                                       RKGS1870
      X=X-H                                                             RKGS1880
      H=.5*H                                                            RKGS1890
      DO 19 I=1,NDIM                                                    RKGS1900
      Y(I)=AUX(1,I)                                                     RKGS1910
      DERY(I)=AUX(2,I)                                                  RKGS1920
   19 AUX(6,I)=AUX(3,I)                                                 RKGS1930
      GOTO 9                                                            RKGS1940
C                                                                       RKGS1950
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE                   RKGS1960
   20 IMOD=ISTEP/2                                                      RKGS1970
      IF(ISTEP-IMOD-IMOD)21,23,21                                       RKGS1980
   21 CALL FCT(X,Y,DERY)                                                RKGS1990
      DO 22 I=1,NDIM                                                    RKGS2000
      AUX(5,I)=Y(I)                                                     RKGS2010
   22 AUX(7,I)=DERY(I)                                                  RKGS2020
      GOTO 9                                                            RKGS2030
C                                                                       RKGS2040
C     COMPUTATION OF TEST VALUE DELT                                    RKGS2050
   23 DELT=0.                                                           RKGS2060
      DO 24 I=1,NDIM                                                    RKGS2070
   24 DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))                             RKGS2080
      IF(DELT-PRMT(4))28,28,25                                          RKGS2090
C                                                                       RKGS2100
C     ERROR IS TOO GREAT                                                RKGS2110
   25 IF(IHLF-10)26,36,36                                               RKGS2120
   26 DO 27 I=1,NDIM                                                    RKGS2130
   27 AUX(4,I)=AUX(5,I)                                                 RKGS2140
      ISTEP=ISTEP+ISTEP-4                                               RKGS2150
      X=X-H                                                             RKGS2160
      IEND=0                                                            RKGS2170
      GOTO 18                                                           RKGS2180
C                                                                       RKGS2190
C     RESULT VALUES ARE GOOD                                            RKGS2200
   28 CALL FCT(X,Y,DERY)                                                RKGS2210
      DO 29 I=1,NDIM                                                    RKGS2220
      AUX(1,I)=Y(I)                                                     RKGS2230
      AUX(2,I)=DERY(I)                                                  RKGS2240
      AUX(3,I)=AUX(6,I)                                                 RKGS2250
```

```
      Y(I)=AUX(5,I)                                                    RKGS2260
   29 DERY(I)=AUX(7,I)                                                 RKGS2270
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)                             RKGS2280
      IF(PRMT(5))40,30,40                                              RKGS2290
   30 DO 31 I=1,NDIM                                                   RKGS2300
      Y(I)=AUX(1,I)                                                    RKGS2310
   31 DERY(I)=AUX(2,I)                                                 RKGS2320
      IREC=IHLF                                                        RKGS2330
      IF(IEND)32,32,39                                                 RKGS2340
C                                                                      RKGS2350
C     INCREMENT GETS DOUBLED                                           RKGS2360
   32 IHLF=IHLF-1                                                      RKGS2370
      ISTEP=ISTEP/2                                                    RKGS2380
      H=H+H                                                            RKGS2390
      IF(IHLF)4,33,33                                                  RKGS2400
   33 IMOD=ISTEP/2                                                     RKGS2410
      IF(ISTEP-IMOD-IMOD)4,34,4                                        RKGS2420
   34 IF(DELT-.02*PRMT(4))35,35,4                                      RKGS2430
   35 IHLF=IHLF-1                                                      RKGS2440
      ISTEP=ISTEP/2                                                    RKGS2450
      H=H+H                                                            RKGS2460
      GOTO 4                                                           RKGS2470
C                                                                      RKGS2480
C                                                                      RKGS2490
C     RETURNS TO CALLING PROGRAM                                       RKGS2500
   36 IHLF=11                                                          RKGS2510
      CALL FCT(X,Y,DERY)                                               RKGS2520
      GOTO 39                                                          RKGS2530
   37 IHLF=12                                                          RKGS2540
      GOTO 39                                                          RKGS2550
   38 IHLF=13                                                          RKGS2560
   39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)                               RKGS2570
   40 RETURN                                                           RKGS2580
      END                                                              RKGS2590
```

APPENDIX C

C.0 ANALOG COMPUTER SIMULATION

The analog computer program required to solve the three degree-of-freedom model is shown in Figure C-1. The diagram shown is simplified over that actually used, in that it does not contain scaling or gain factor information, nor does it show certain inverter amplifiers required by the non-linear equipment. Standard analog techniques are used throughout the program, except perhaps where multiplier feedback was used around the $\ddot{\theta}$ and $\ddot{\phi}$ summers in preference to resorting to the use of less accurate division circuitry. The generation and control of the firing pulse F(t) was achieved by use of an internal timer, a logic gate (G), and a delay flop (P) to activate two integrators separately from the rest of the computer equipment. The timer controlled the firing frequency and the delay-flop controlled the duration of the F(t) function. Equations 2-9 required linearization before they could be implemented on the analog computer. Small angle approximations were made and certain higher-order terms were removed. The final set of differential equations solved was as follows:

$$m_r \ddot{x} = m_r L \ddot{\theta} + m_r x \dot{\theta}^2 + m_r x \dot{\phi}^2 - (c_x \dot{x} + k_x \{x - x_o\}) - F(t)$$

$$[I_{cm} + I_r + M r_{cm}^2 + m_r (x^2 + L^2)] \ddot{\theta} = m_r L \ddot{x} - (I_r + m_r x^2) \ddot{\phi}$$

$$- m_r L x \dot{\phi}^2 - c_\theta \dot{\theta} - k_\theta \theta - m_r g (x - x_o) + L F(t)$$

$$(I_r + m_r x^2) \ddot{\phi} = - (I_r + m_r x^2) \ddot{\theta} + m_r L x \dot{\theta}^2 - c_\phi \dot{\phi} - k_\phi \phi - m_r g (x - x_o) + \delta F(t)$$

The main variables of interest (those monitored in actual tests) are output from the computer solution as F_{RECOIL}, the force felt on the gunners shoulder, and the total pitch angle $(\theta + \phi)$.

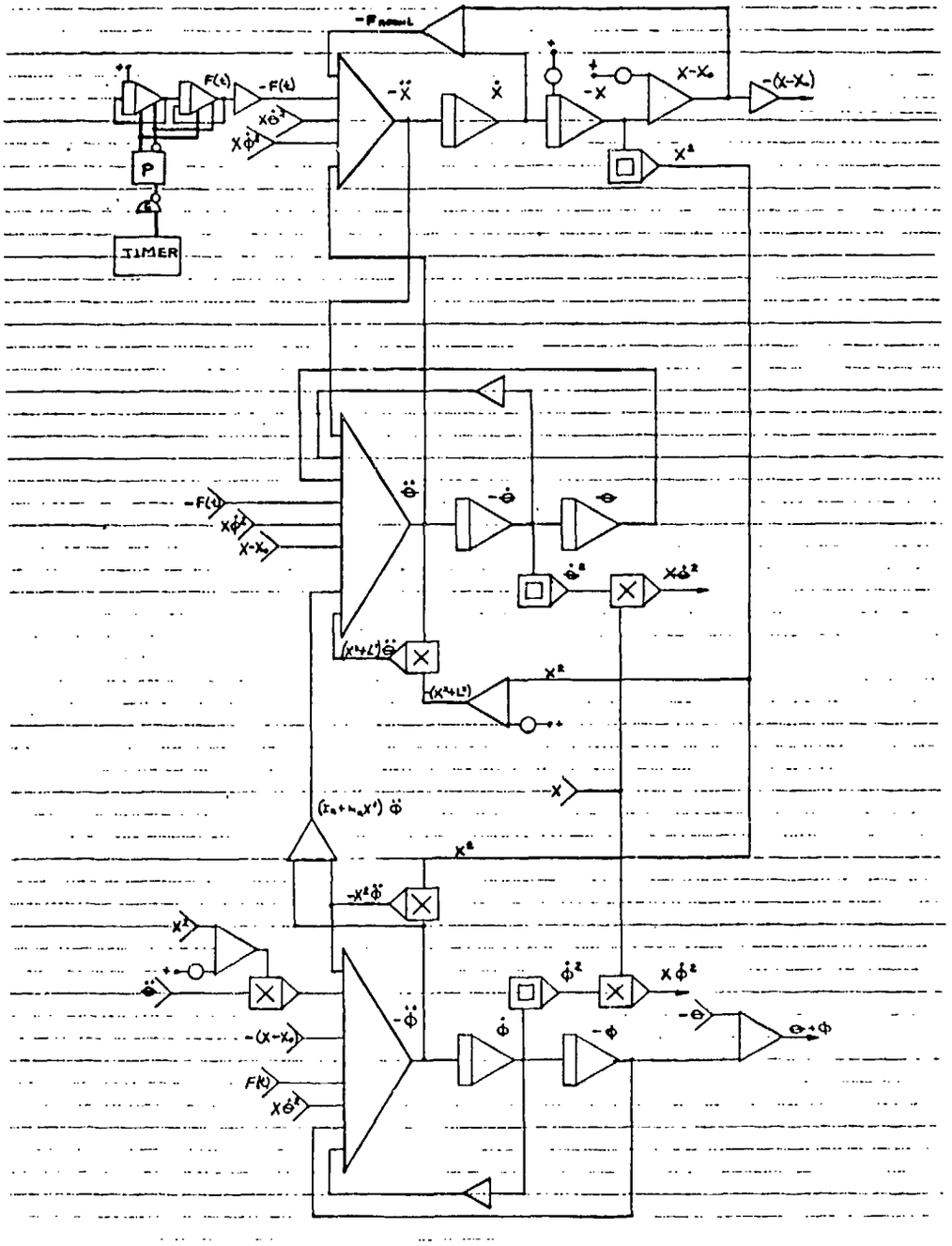

Figure C-1 Simplified Analog Computer Diagram of Man-Weapon Interaction Equations

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A study was conducted of the man-weapon interaction force relationship to define the parameters to be incorportated in the design of a universal small arms test fixture, that simulates man as a flexible mount. This report contains the results of Phase I of the program, which is to provide the engineering data and preliminary drawings required for the fabrication of a prototype test fixture. Phase II will provide for the manufacture and evaluation of the test fixture. To demonstrate the feasibility of designing a

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universal mount fixture, a three degree of freedom mathematical model was developed that simulates the man and weapon as a coupled dynamical system. A sensitivity analysis was performed on both analog and digital computers to determine which of the system parameters are critical to the design of the mount and to obtain bounds on them. The mathematical simulation was supported by an extensive test firing program, involving shooters of various sizes firing the M16 rifle and the M79 and M203 grenade launchers. In these tests the shoulder reaction force and the dynamical motions of the man-weapon system were recorded. Information provided by the tests was used to estimate some of the critical parameters of the system and to provide a means for validating the model predictions. Results of this study indicate the feasibility of designing a universal small arms mount based on a two degree of freedom concept that would be capable of testing a variety of small arm weapons.

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